IS INFLATION VOLATILITY CORRELATED FOR THE US AND CANADA?

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ABSTRACT

This paper investigates the behavior of inflation over the recent past for the US and Canada with primary focus on volatility, not the level of inflation. Over the past 35 years, the inflation rates for this pair of nations have shown periods of tranquility as well as periods of volatility. Recent evidence suggests that inflation, after a period tranquility during the 1990s for the US and for Canada, became more volatile early in the new century (perhaps even as early as 1999)—prior to the volatility in the energy and food sectors. Evidence of the volatility is presented and is modeled with relatively simple autoregressive conditional heteroskedasticity (ARCH) models. The resulting series on volatility are then compared across the US and Canada.

INTRODUCTION

The autoregressive conditional heteroskedasticity (ARCH) model was developed by Robert Engle to explain volatility "clustering," that is, periods in which the variance of a time series is tranquil and other periods in which the variance of the series is more volatile. The ARCH model and its extension, generalized ARCH (GARCH), have been applied to numerous economic and financial series. These models are important in identifying periods of volatility and they also aid in producing more realistic interval forecasts.

DATA, METHOD, RESULTS

For this project monthly measures of the Consumer Price Index (CPI) for the period January 1974 to December 2009 were collected for both nations. The series are for "headline" inflation rates, not core inflation. The start date was chosen to coincide with the end of the Bretton Woods fixed exchange rate era. The measure of inflation is the monthly log difference in the CPI at annual rates. Those series are shown in Figure 1 for the US and in Figure 2 for Canada.

Casual observation of Figures 1 and 2 suggests that inflation was more volatile in the late 1970s and again in the 2000s for both nations, though the volatility in the 2000s seems somewhat greater for the US (note that the left-hand scales are not the same between the graphs). Periods of tranquility were evident in the 1990s for both nations. Canada also experienced a more tranquil period than the US from about 1984 until 1990. The spike in Canadian inflation evident in 1991 was distorted by the institution of the federal GST (Goods and Services) tax.

It is well known that simple inspection of the variance of a series can be misleading when the series is autocorrelated. To correct for this, an autoregressive model is fit to the inflation rate. The lags are chosen using standard penalized likelihood model selection criteria. The form of the autoregressive model can be represented as follows:

$$INFL_t = a_0 + \sum_{i=1}^{p} b_i INFL_{t-i} + e_t \qquad (1)$$

Figure 1: Monthly US Inflation at Annualized Rates

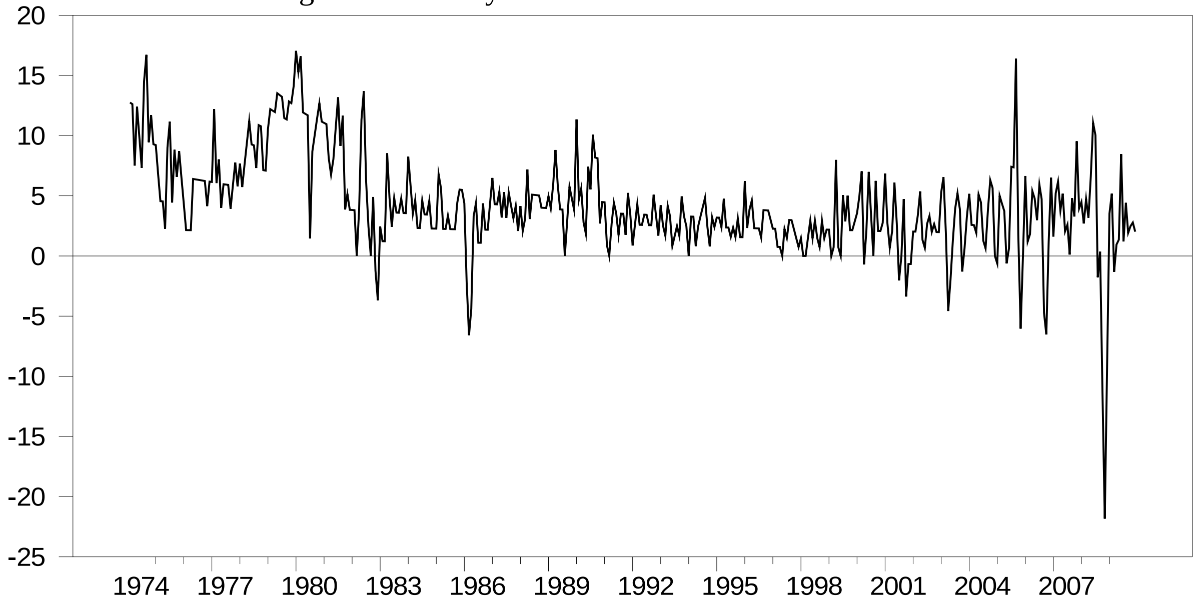
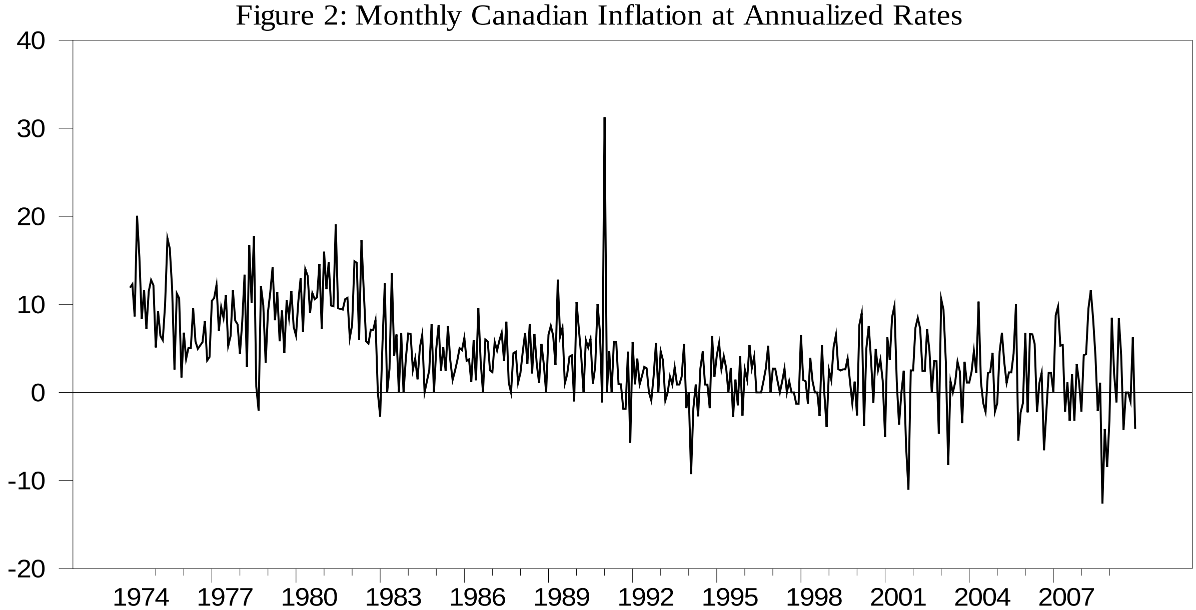


Figure 2: Monthly Canadian Inflation at Annualized Rates



where *INFL* is annualized monthly inflation, t indexes time, e_t is a white noise disturbance term and the $b_i \; (i = 1,..., p)$ are the lag coefficients, and p indicates the order of the lags. The two standard penalized likelihood selection criteria are the Akaike information criterion *(AIC)* and the Schwarz information criterion *(SIC)* represented as follows:

$$AIC = (2k / T) + \log(\sigma) \tag{2}$$

$$SIC = [k \log(T) / T] + \log(\sigma), \tag{3}$$

where k is the total number of estimated coefficients in the equation, T is the number of usable observations, and σ is the scalar estimate of the variance of the equation's disturbance term. If the *AIC* and the *SIC* differ on the number of lags, each indicated model was estimated, with evidence presented here for the most parsimonious model. The *SIC* chooses p = 2 for the US and p = 12 for Canada. Additional analyses are based on those estimated models.

Figure 3: Residuals From the AR Model for the US

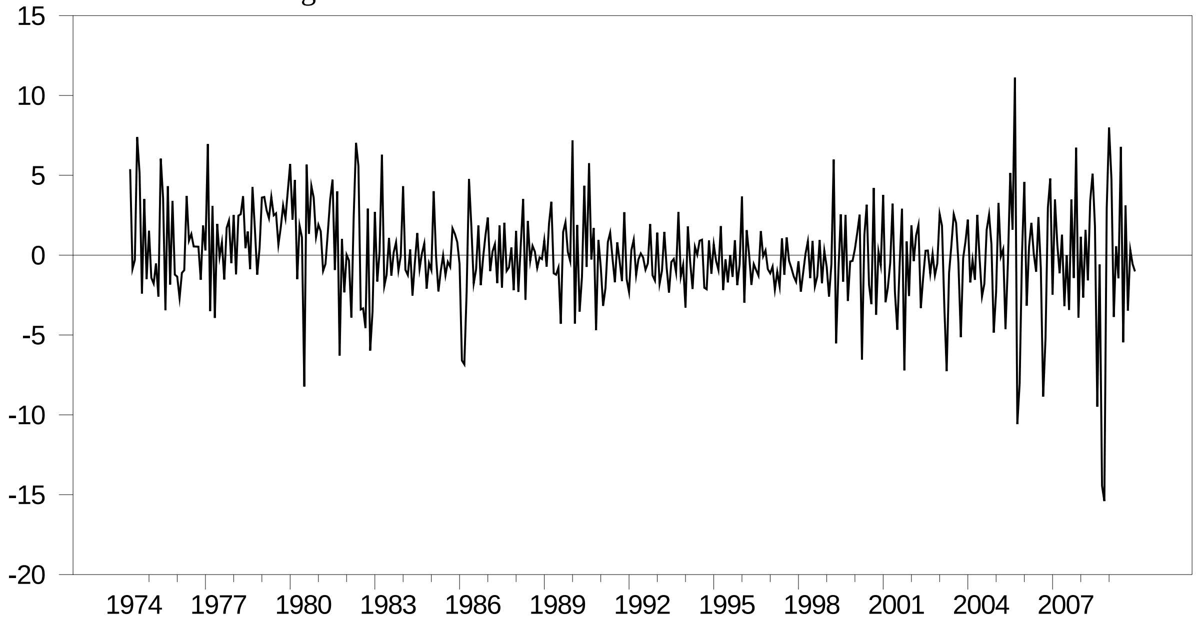
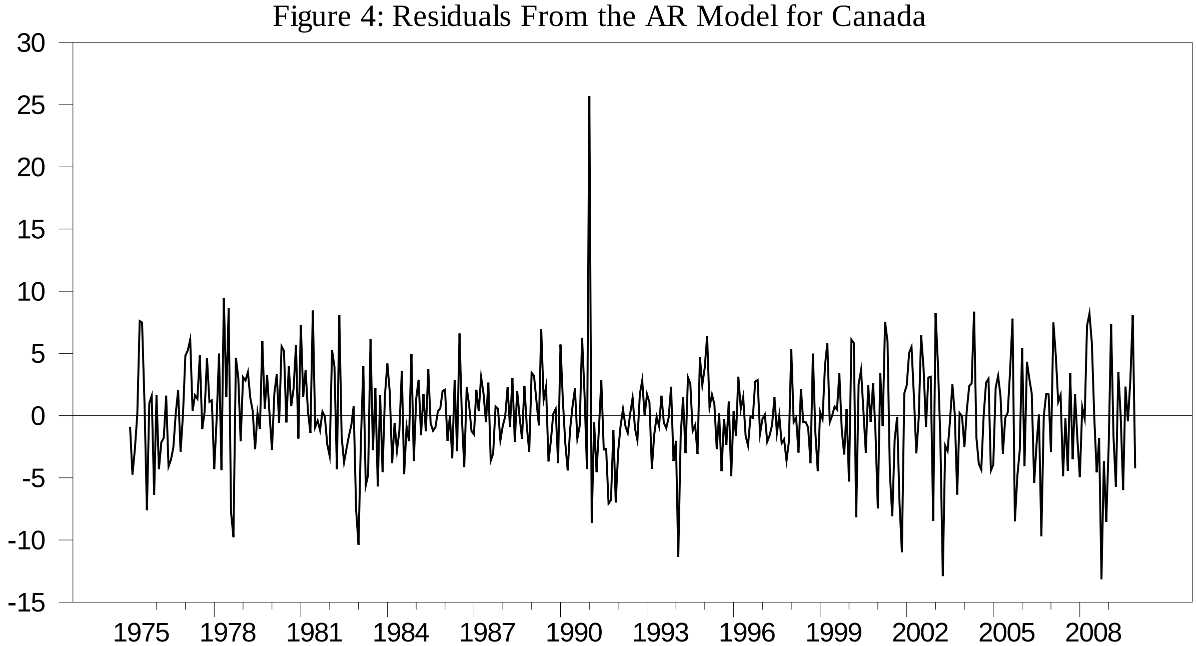


Figure 4: Residuals From the AR Model for Canada



The residual series from the autoregressive models are presented in Figures 3 and 4. These series are often considered a close approximation of the volatility of the series, since the models control for autocorrelation. Figure 3 suggests the same evident patterns of volatility for the US. Figure 4, for Canada, is less clear. An increase in volatility seems to be evident after 1999 for Canada, but it appears less pronounced than that for the US.

Figure 5: Squared Residuals From the AR Model for the US

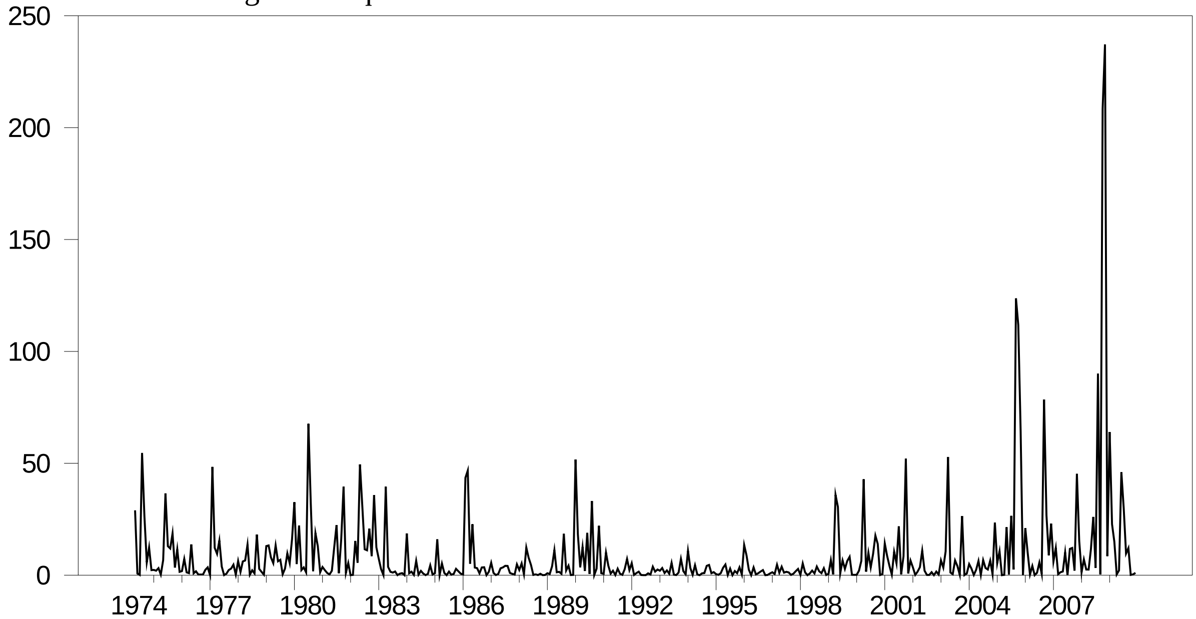
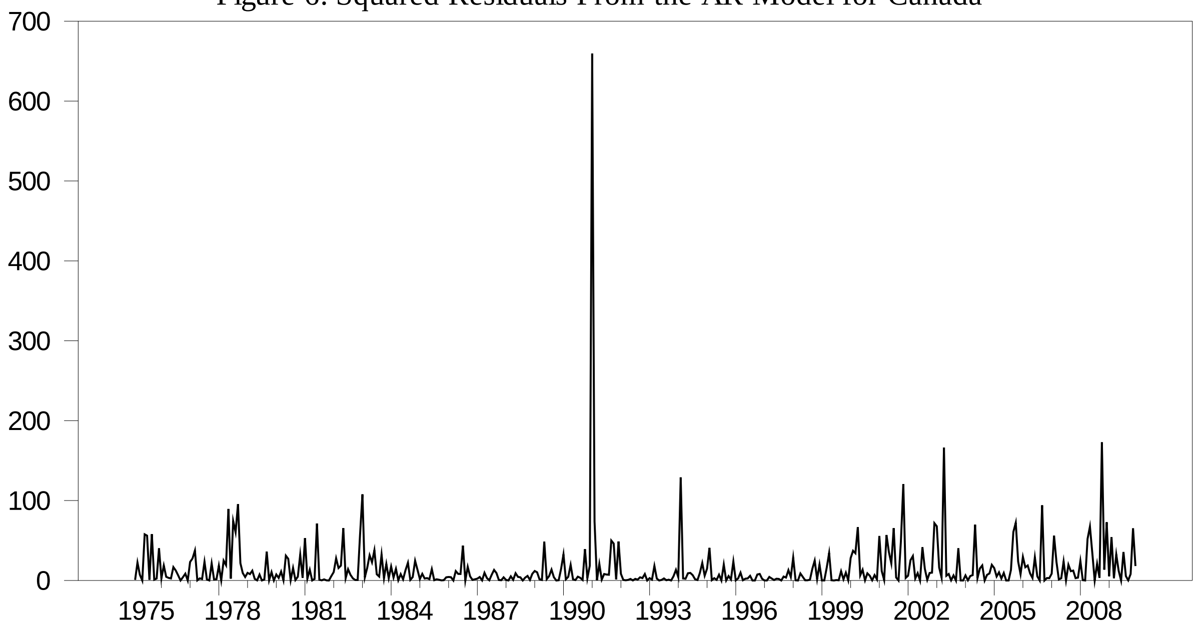


Figure 6: Squared Residuals From the AR Model for Canada



Examining the squared residuals in Figures 5 and 6 may be more revealing. Figure 5 shows a definite pattern of clustering of volatility for the US, with evident tranquility in the 1990s, followed by a persistently greater volatility following 1999. For Canada, if the spike in 1991 is ignored, the period from 1984 until 1999 would appear to be one of tranquility, followed by an increase in volatility thereafter.

Testing for volatility is usually accomplished by analysis of the squared residuals from an autoregressive model, such as those depicted in Figures 5 and 6. The reasoning for testing the squared residuals is simple. The residuals from the autoregressive model (see Figures 3 and 4) will be serially uncorrelated as a result of the autoregressive lag fit. Those residuals are, however, not independent. Small (in absolute value) residuals are likely to be followed by additional small residuals, and large residuals are likely followed by other large residuals—that is the meaning of volatility clustering.

To test for ARCH errors, a second regression is run:

$$e_t^2 = c_0 + \sum_{i=1}^{p} d_i e_{t-i}^2 + v_t \tag{4}$$

Where e_t^2 represents the squared residuals from equation 1, and the $d_i\ (i = 1,\ldots, p)$ are lag coefficients and p again indicates the order of the lags. If there are no *ARCH* effects, then equation 4 will have little explanatory power, i.e., R^2 will be very low. The existence of *ARCH* effects can be tested in two ways. First with a sample of T residuals, TR^2 is distributed as χ^2 with p degrees of freedom. Alternatively, an F-test that all d_i coefficients are jointly zero will also indicate whether or not *ARCH* effects are present. The *SIC* chooses 2 lags for equation 4 for the US and only 1 lag for Canada.

The estimated equations for (4) are:

US:

$$\hat{e}_t^2 = 4.94 + 0.35\hat{e}_{t-1}^2 + 0.08\hat{e}_{t-2}^2 \tag{4′}$$

$R^2 = 0.152$
$T = 426$

The null hypothesis of no *ARCH* effects can be written:

H_0: $d_1 = d_2 = 0$ (there are no *ARCH* effects)
H_1: some $d_i \neq 0$ (there are *ARCH* effects)

As expected, the null hypothesis is rejected resoundingly for either the χ^2 test (χ^2 = 66.46, p-value = 0.0000), or the F-test ($F_{(df = 2,423)}$ = 39.10, p-value = 0.0000). We conclude that the process of inflation for the US is subject to *ARCH* effects. Thus we have confirming statistical and visual evidence that small squared residuals tend to be followed by small squared residuals, and large squared residuals are more often followed by other large squared residuals.

Canada:

$$\hat{e}_t^2 = 14.39 + 0.09\hat{e}_{t-1}^2 \tag{4″}$$

$R^2 = 0.0054$
$T = 418$

Here finding in favor of *ARCH* is a little more ambiguous. Because the equation contains only one lag for squared error series, the above tests are equivalent to the t-test on the slope coefficient. That t-score is 1.8 with a p-value of 0.072. A tentative conclusion can be offered: *ARCH* effects likely exist for Canadian inflation but are much weaker than in the case of the US.

OTHER RESULTS

The *ARCH* errors model is typically estimated simultaneously with the autoregressive model of inflation by maximum likelihood methods. That estimation also yields an estimate of the variance of the series, typically known as the *h* series. Again choosing the same values *p* for the autoregressive presentation for inflation, and for the variance of the series of each nation, we present the portion of the equation that represents the variance (here, *h*) of the inflation series (here we are less interested in the autoregressive parameters of the estimate of inflation, since many, many, alternative inflation forecasting models are possible):

$$US\,h_t = 1.08 + \underset{(3.56)}{0.20}\,\hat{e}_{t-1}^2 + \underset{(4.22)}{0.26}\,\hat{e}_{t-2}^2 \tag{5}$$

$$Can\,h_t = 0.43 + \underset{(2.60)}{0.25}\,\hat{e}_{t-1}^2 \tag{6}$$

Where *h* is the estimated conditional variance in inflation and the numbers in parentheses are t-statistics. Equation 6 suggests stronger *ARCH* effects for Canada than did the prior section.

To summarize the results of this section, we find in favor of *ARCH* effects for the inflation series of both nations. The statistical and visual evidence are very clear for the US, weaker for Canada.

TWO TESTS OF VOLATILITY RELATIONSHIPS ACROSS THE US AND CANADA

There are at least two potential tests for the correlation of volatility across nations. First, the hypothesis that the residual series are correlated is entertained, and second, a test for dependence in the squared residual series is run. The results of such tests will lead to conclusions regarding whether shocks to the inflation series which result in changes in volatility are coordinated across these two nations, and the lag/lead nature of such coordination, should it exist.

Recall that the residual series derived from models represented by equation 1 above are not autocorrelated. The series may be, however, correlated with the residual series from the "other" nation. For example, the residual series from the US, though not autocorrelated, may be correlated with the residual series from Canada. This hypothesis is tested with each nation's residual series as the dependent variable. Here I choose arbitrarily leads and lags of a maximum of 3. For example, with the US residual series as the dependent variable, the representative regression is:

$$\hat{e}_t^{(US)} = c_0 + \sum_{i=-3}^{+3} d_i \hat{e}_{t-i}^{(C)} + v_t\ , \tag{7}$$

with the US and C reversed for the second regression. The result of those two estimations can be described as follows. First, the residual series is most strongly correlated at the contemporaneous month, that is, at lag = 0, and as expected the relationship is positive. Second, the US residual series is correlated with the residuals for Canada at lead = 1. That is, US volatility leads Canadian volatility. That result is confirmed for the case with the Canadian residual series serving as the dependent variable, where at lag = 1, the US residual series is related to Canadian volatility. No other lag (or lead) is statistically significant. These results indicate that inflation volatility, measured by the residual series from separate autoregressions, is correlated between the US and Canada, and further that US volatility leads Canadian volatility.

Further evidence on volatility dependence across the US and Canada can be tested via traditional vector autoregressions (VARs) for the squared residual series. Equations 4′ and 4″ already established that volatility is subject to clustering for both the US and Canada, thus accounting for "own" lags will allow testing for Granger [6] causality *across* the two nations via lagged variance series of the other nation. Such a formulation can be represented as:

$$\hat{e}_t^{2(US)} = a_0 + \sum_{i=1}^{p} b_i \hat{e}_{t-i}^{2(US)} + \sum_{i=1}^{p} c_i \hat{e}_{t-i}^{2(C)} + v_t \ , \qquad (8)$$

and, of course, the Canadian squared residuals $\hat{e}_t^{2(C)}$ also serves as the left-hand side variable. The test for Granger causality is an F-test that the b_i coefficients are zero, and another F-test that the c_i coefficients are zero. The AIC and the SIC differ on the number of lags *(p)*, with the SIC choosing $p = 1$, and the AIC choosing $p = 12$. Each of those estimations yields the same general conclusion. If the dependent variable is $\hat{e}_t^{2(US)}$, then lagged US variance causes (in the sense of Granger) US variance in the current time period, but Canadian lagged variance does not. In the case of Canadian variance in inflation, neither lagged series is important in predicting the current variance in inflation.

Table I: F-tests for $p = 12$

Dependent Variable	Explanatory Variables	
	Lagged U.S. variance	Lagged Canadian variance
U.S. variance	7.982* (0.00)	0.577 (0.86)
Canadian variance	1.322 (0.20)	0.616 (0.828)

(p-values in parentheses)

Table II: F-tests for $p = 1$

Dependent Variable	Explanatory Variables	
	Lagged U.S. variance	Lagged Canadian variance
U.S. variance	67.02* (0.00)	2.53 (0.111)
Canadian variance	0.828 (0.364)	2.54 (0.112)

(p-values in parentheses)

The more parsimonious estimations in Table II give similar results, though the lagged values of the Canadian series come closer to meeting tests for statistical significance.

ECONOMICS OF INFLATION VOLATILITY ACROSS NATIONS

Economists have long believed that a stable inflation rate is one of the keys to a smoothly functioning macro economy (see [5], for example). It follows that inflation volatility can be a source of economic instability. The possible sources of inflation instability include exogenous price shocks, speculative bubbles, monetary policy, and even openness in trade and capital flows. Why then might inflation volatility be synchronized across nations? First exogenous price shocks—for example to food and energy—may be felt across nations, though the effects on the individual economies may (of course) differ. The same can be argued for speculative bubbles. Second, monetary policies may be coordinated across nations, with the result that periods of tranquility (and volatility) are similar. Third, shocks that originate in one nation may affect inflation in other nations through trade and capital flows as well.

CONCLUSIONS

This research finds in favor of modeling inflation in the US and Canada as *ARCH* processes, though the evidence is stronger for the US. The cross country tests for correlation of inflation variance yields two important results. First, US and Canadian inflation volatility, measured as the residuals from autoregressive representations, are correlated in the contemporaneous month and there is evidence that US volatility leads Canadian volatility by one month. Second, when volatility is measured as the squared residuals from the autoregressive representation, the evidence for cross country Granger causality is weak. The latter would suggest that prediction of volatility for, say, the US would not be aided by modeling lags of Canadian volatility.

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