REVIEW AND REFORMULATION OF THE MAXIMUM AVAILABILITY LOCATION PROBLEM

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ABSTRACT

In this paper we show that the maximum availability location problem (MALP) can be reformulated with fewer variables and constraints. Our computational experiments demonstrate that the reformulated MALP (rMALP) solves much faster and also lends itself to more efficient heuristic development.
Keywords: Location problems; emergency medical services

INTRODUCTION

The analysis of the location of Emergency Medical Response (EMR) vehicles has been an ongoing topic of research for over 40 years. Numerous mathematical models have been created to help optimize coverage by utilizing a limited number of resources [1, 6]. Decisions have to be taken in several areas including the location of the Emergency Response Vehicles, the number of resources to be used and the order in which the vehicles are dispatched.

Due to limited resources in many instances response within an acceptable time cannot be guaranteed for all calls within the system. In this regard a number of trade-offs need to be made. For example minimizing overall response times will result in a smaller coverage area. Depending on the area of emphasis, efficacy of a system can be measured in a number of different ways:

- The total response time is minimized
- The total area covered is maximized
- The maximum time taken to respond to a call is minimized
- The number of calls which are covered within an acceptable time is maximized
- The total response time is minimized while ensuring that all calls are covered within an acceptable time

In the literature models are typically classified as deterministic or probabilistic and furthermore static or dynamic. Deterministic models are prescriptive in nature and used to find the optimal location of ambulances to minimize or maximize an objective. Probabilistic models acknowledge the possibility that a given ambulance may not be available when it is called. These types of models provide a way to model uncertainty by either using a queuing framework or via a mathematical programming approach. Several probabilistic models have been used to determine optimal location points for the EMR vehicles. ReVelle and Hogan proposed the Maximum Availability Location Problem [7]. This is a novel extension of the Maximum Coverage Location Problem introduced by Church and ReVelle in 1974 [3]. The MALP addressed one of the primary shortcomings of the several deterministic models proposed at the time where no allowance was made for the probability that EMS vehicles can be busy. It makes explicit provision for the non availability of servers (servers being out on call). In our proposed reformulation of the MALP we show how it can be made more compact and faster to solve with off-the shelf solvers.

THE MAXIMUM AVAILABILITY LOCATION PROBLEM

The classic Maximum Covering Location Problem (MCLP) developed by Church and ReVelle [3] did not address EMR vehicle unavailability. ReVelle and Hogan extended the MCLP to explicitly consider the possibility that when requested an EMR vehicle (e.g., ambulance) may be busy serving an earlier emergency. Therefore they defined the Maximum Availability Location Problem objective as to maximize the population the servers can cover (within a target response time) with a reliability of α. To model the congestion in the system ReVelle and Hogan use the busy fraction defined in the Maximum Expected Coverage Location Problem introduced by Daskin [4, 5]. Let,

x_i = number of servers positioned in node (district, zone) i

n = the number of nodes in the system

h_j = demand at node j

m = number of ambulances available

$\bar{t}$ = average service time

$$a_{ij} = \begin{cases} 1 \text{ if node } j \text{ is covered by server at node } i \text{ (within response time target)} \\ 0 \text{ if not} \end{cases}$$

Then the busy fraction can be estimated by:

$$p = \frac{\bar{t}\sum_{j=1}^{n} h_j}{24\sum_{i=1}^{n} x_i} = \frac{\bar{t}\sum_{j=1}^{n} h_j}{24m} \qquad (1)$$

Once the busy fraction has been determined chance constraints formulated by Charnes and Cooper are used [2]. The chance constraint is

$$1 - p^{\sum_{i=1}^{n} a_{ij} x_i} \geq \alpha \qquad (2)$$

Where α is desired coverage reliability. ReVelle and Hogan solve for the number of servers (ambulances) required to meet (2) above by:

$$\sum_{i=1}^{n} a_{ij} x_i \geq b \qquad (3)$$

$$b = \left\lceil \frac{\log(1-\alpha)}{\log p} \right\rceil \qquad (4)$$

Therefore each demand area will require at least *b* servers in order to attain the required level of coverage with reliability.
Let,

$$y_{jb} = \begin{cases} 1 \text{ if } b \text{ servers cover node } j \\ 0 \text{ if not} \end{cases}$$

The objective function in the MALP is to maximize the total demand covered by at least an α level of reliability.

$$\textit{Maximize: } \sum_{j=1}^{n} h_j y_{jb} \qquad (5)$$

Subject to:

$$\sum_{i=1}^{n} a_{ij} x_i \geq \sum_{k=1}^{b} y_{jk} \quad \forall j \qquad (6)$$

$$y_{jk} \leq y_{j,k-1} \quad \forall j \qquad (7)$$

$$\sum_{i=1}^{n} x_i \leq m \qquad (8)$$

$$y_{jk}, x_i \in \{0,1\} \quad \forall i, j \qquad (9)$$

Constraint (6) determines if a node is covered by at least b servers. Constraint (7) ensures that a node is first covered once before it is covered twice or thrice and so on. Constraint (8) ensures that total number of vehicles used is not greater than the total number of vehicles available.

Reformulation of MALP

Instead of utilizing y_{jk} we let

$$y_j = \begin{cases} 1 \text{ if node j is covered at least b times} \\ 0 \text{ if not} \end{cases}$$

Therefore the objective function becomes,

$$\text{Maximize } \sum_{j=1}^{n} h_j y_j \qquad (10)$$

We now rewrite constraint (6) as follows,

$$\sum_{i=1}^{n} a_{ij} x_i \geq b y_j \quad \forall j \qquad (11)$$

$$y_j \in \{0,1\} \quad \forall j \qquad (12)$$

Constraint (10) ensures that node *j* is deemed covered ($y_j = 1$) only if it is within coverage distance of the required number of vehicles which is denoted by *b* and the objective function (10) tallies only the demand (population) covered by at least b times. By changing constraint (6) we eliminated the need for constraint (7).
In the original MALP the number of nodes j leads to (2j + 1) constraints and the numbers of nodes j and i along with the required number of ambulances b gives us (i + jb) variables. In the reformulated model the number of variables is (j + 1) and the number of constraints (j+1). The reformulation thus saves j number of constraints and (b-1)j number of variables when b>1.

RESULTS AND CONCLUSIONS

In order to compare the original MALP and the revised model we generated a region which is 1024 sq miles in size and is divided in to 256 zones. Each zone is a square, 4 sq miles in size (2 miles by 2 miles). We randomly generated uniformly distributed call (demand) rates for each of the zones. Ten different sets of data were generated. We then proceeded to apply both models to the generated data at different levels of b, resulting in 50 problems. We used LINDO 6.1 to solve the problem.

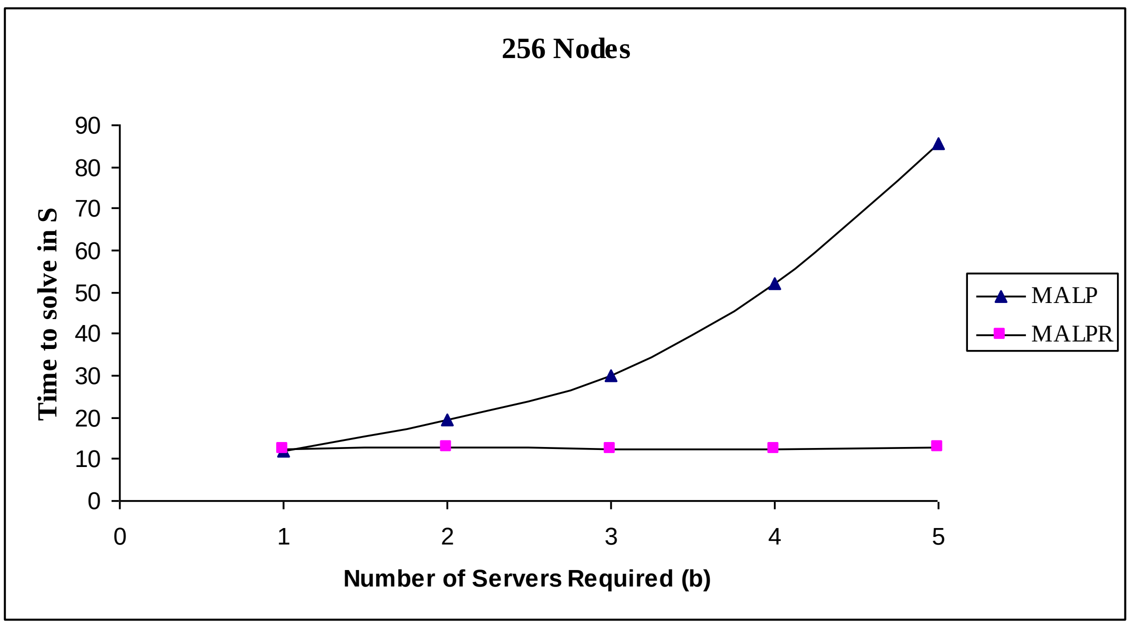

Figure 1. Average CPU times for MALP and rMALP.

The results shown is Figure 1 indicate that the time to solve for the original MALP increases exponentially as the number of required servers increase. The revised MALP utilizes the same number of variables and constraints for all levels of *b* therefore the average time to solve remains consistent. For problems with a larger number of zones using the revised MALP would result in a significant saving of time without any loss of optimality. To further test this conjecture we are developing large scale problem instances with zones up to 1,024 and using non-uniform distributions to randomly generate call volumes across the (hypothetical) regions.

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