The Green Single Vehicle Routing Problem

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ABSTRACT

This paper introduces a new version of the traditional Vehicle Routing Problem (VRP). The objective function of the VRP is to minimize the total distance traveled by a set of vehicles serving a set of customers. This research proposed a new objective function that minimizes Ton-Miles. Fuel consumptions and consequently CO_2 emissions are a function of Ton-Miles instead of total distance travelled by the vehicles. More exactly, this research presents the case for a single vehicle. This work includes a mathematical formulation of the problem; a small instance that illustrate the problem, and a local search technique that can be used in future approximation algorithms. The new problem is called the Green Single Vehicle Routing Problem (GSVRP).

INTRODUCTION

The vehicle routing problem (VRP) is the scheduling of a set of vehicles that serves a group of customers such that the total distance traveled by the vehicles is minimized. The VRP was first introduced by [1] and have been widely studied in the literature. In fact, the VRP was proved to be NP-Hard by [2]. It means that there is no algorithm that guarantee solution optimality for larger problem instances in acceptable computational times.

The VRP has applications in the transportation and logistics fields. Moreover, any company such as FedEx and UPS that scheduled vehicles to serve customers need to solve VRP routinely. There are different versions of the problem according to the assumption considered. For example, there are VRP that allow customers to be visited only once, while others allow customers to be visited more than once. Also, there are versions of the problem that only pickup or deliver products to customers, while others allow simultaneously pickup and delivery. For a detailed description of the different versions of the VRP the reader is referred to [3] [4] and [5].

As mentioned above, the most common objective function among VRP is to minimize total distance travelled by the vehicles. An alternative and more environmentally friendly objective for the problem is to minimize fuel consumptions or CO_2 emissions. Therefore, minimizing total Ton-Miles travelled by the vehicle would minimize fuel consumption and consequently CO_2 emissions. The purpose of this paper is to introduce a new version of the VRP that minimizes Ton-Miles. Moreover, the problem presented in this research considers only one vehicle and is called the Green Single Vehicle Routing Problem

(GSVRP). In addition, it is important to mention that the single VRP is the same as the Travelling Salesman Problem (TSP), which was proved NP-Hard by [2]. Since the GSVRP is a generalization of the single VRP and the TSP, the GSVRP is also computationally difficult. Therefore, approximation algorithms such as Simulated Annealing and Tabu Search are required to solve large instances in acceptable computational time.

The remaining of this paper is as follows: Section 2 formally introduces a mathematical formulation for the GSVRP. Section 3 illustrates the problem using a small problem instance. Section 4 illustrates a local search algorithm. Finally, section 5 summarizes this work and proposes future avenues of research.

PROBLEM DESCRIPTION

Properly, the GSVRP can be defined as follows: Given a set of customers that need to be served by one vehicle, the GSVRP is to find the route that visits each customer exactly once and return the vehicle to the depot such that the total Ton-Miles accounted is minimized.

Mathematical Formulation

The mathematical formulation for the GSVRP presented below is build over the traditional formulation of the TSP. Following are the indexes, parameters, variables, objective function, and constraints of the mixed linear integer programming for the GSVRP.

Indexes:

i, *j* Locations: *i*, j = 0, 2, ..., L; where 0 represents the location of the depot.

Parameters:

- d_{ij} Distance between locations *i* and *j*.
- q_i Customer *j* demand.
- *W* Vehicle weight including all the cargo.

Variables

- x_{ij}^k : 1 if vehicle serves location *j* immediately after serving location *i*. 0 otherwise.
- y_{ij} Weight of vehicle traveling between locations *i* and *j*.
- u_i Arbitrary real variable.

Objective function:

$$Min\sum_{i=0}^{L}\sum_{j=0}^{L}d_{ij}y_{ij} \qquad i \neq j$$
(1)

Subject to

$$\sum_{i=0}^{L} x_{ij} = 1 \qquad \forall j \neq i$$
⁽²⁾

$$\sum_{i=1}^{L} x_{ij} = 1 \qquad \forall i \neq j \tag{3}$$

$$\sum_{j=0}^{L} y_{0j} = W$$
 (4)

$$\sum_{i=1}^{L} y_{ij} - \sum_{i=1}^{L} y_{ji} = q_j \qquad \forall j, i \neq j$$
⁽⁵⁾

$$y_{ij} - Wx_{ij} \le 0 \qquad \forall i, j, i \ne j \tag{6}$$

$$u_i - u_j + Lx_{ij} \le L - 1 \tag{7}$$

$$x_{ij} \in [0,1] \qquad \forall i, j, i \neq j \tag{8}$$

$$u_i \ge 0 \qquad \forall i \tag{9}$$

Objective function (1) minimizes total ton-miles. Constrain set (2) ensures that the vehicle arrives to each location once. Constraint set (3) ensures that the vehicle leaves each location once. Constraint set (4) ensures that the initial weight of the vehicle includes its own weight and the weight associated to customer demands. Constraint set (5) guarantees flow conservation. Constraint set (6) ensures that vehicle weights are in concordance with the tour order. Constraint set (7) eliminates sub-tours. Constraint sets (8) and (9) restrict decision variables. Notice that constraint sets (2), (3) and (7) are from the TSP formulation and constraint set (6) link TSP constraints with the additional parts of the GSVRP formulation.

PROBLEM INSTANCE

The GSVRP will be illustrated using a small problem instance. The instance considers a single vehicle with a curb weight (i.e., weight of the empty vehicle) of 8 tons. More over the vehicle will delivery cargo to 5 customers. The customer demands are 1.5 ton for customer 1, 0.50 ton for customer 2, 3.5 ton for customer 3, 3.5 ton for customer 4, and 0.5 ton for customer 5.

Table 1 shows the distance between each pair of locations. Moreover, location 0 represents the vehicle depot.

	0	1	2	3	4	5
0	0	33.5	30.4	22.4	17.1	22.4
1	33.5	0	20.0	21.2	26.9	20.6
2	30.4	20.0	0	22.2	25.0	25.0
3	22.4	21.2	22.2	0	25.8	24.1
4	17.1	29.9	25.0	25.8	0	15.8
5	22.4	20.6	25.0	24.1	15.8	0

Table 1: Distances between locations (location 0 represents the depot)

The optimal solution of the problem instance was obtained by solving the mathematical formulation above with lp_solve 5.5. The optimal solution is showed in the left part of table 2. Notice that the route that minimizes Ton-Miles is $S = \{0, 4, 3, 2, 1, 5, 0\}$, where S shows the sequence in which customers should be visited. The route has Ton-Miles 1405.3 and the total distance travelled by the vehicle is 128.1

miles. On the other hand, when the problem is solved using a TSP mathematical formulation with lp_solve 5.5 (i.e., minimizing the total distance traveled by the vehicle), a different solution $S = \{0, 3, 2, 1, 5, 4, 0\}$ is obtained. Notice that the total number of miles in 10 miles less than the GSVRP, but Ton-Miles is 100.9 higher. The details of the TSP solution are showed in the right side of table 2.

GSVRP Optimal solution					TSP Optimal solution				
Tour	Distance	Demand	Vehicle	Ton-	Tour	Distance	Demand	Vehicle	Ton-
			weign	willes				weign	willes
0					0				
	17.1		17.0	290.7		22.4		17	380.8
4		3.5			3		3.5		
	25.8		13.5	348.3		22.2		13.5	299.7
3		3.5			2		0.5		
	22.2		10.0	222.0		20.0		13.0	260.0
2		0.5			1		1.0		
	20.0		9.5	190.0		20.6		12.0	247.2
1		1.0			5		0.5		
	20.6		8.5	175.1		11.5		11.5	181.7
5		0.5			4		3.5		
	22.4		8.0	179.2		17.1		8.0	136.8
0					0				
TOTAL	128.1			1405.3	TOTAL	118.1			1506.2

Table 2: GSVRP and TSP optimal solutions.

LOCAL SEARCH TECHNIQUE

Since the GSVRP is computationally difficult, approximation algorithms are required for solving large size problem instances. Approximation algorithms such as Tabu Search and Simulated Annealing use Local Search (LS) techniques (LS) in their exploration of the problem solution space. LS require a solution representation and a mechanism to explore neighboring solutions. A solution for the GSVRP can be represented as a vector $S = \{s_0, s_1, ..., s_L, s_{L+1}\}$ showing the order in which customers will be visited. Moreover s_0 and s_{L+1} represent the vehicle depot and can be omitted from the solution representation. For example a solution for the problem can be represented as $S = \{3, 2, 1, 5, 4\}$ where $s_1=3$, $s_2=2$ and so on. Notice that LS needs and initial solution. Commonly, initial solutions are provided by construction algorithms. A simple construction algorithm for the GSVRP is to assign customers in sequential order, that is $S = \{1, 2, 3, 4, 5\}$. Once an initial solution is obtained for LS, the new solution becomes the current solution.

Among the most effective mechanism to explore neighboring solutions is 2^{opt} . A 2^{opt} mechanism exchange two customers in the current solution *S*. For example a 2^{opt} move could exchange customers 3 and 2 in *S* leading to a new neighboring solution $S^* = \{2, 1, 3, 4, 5\}$. Once the move is performed, the objective function value (i.e., Ton-miles) of the solution is obtained. The 2^{opt} mechanism is repeatedly applied to *S* until all possible 2^{opt} moves are evaluated. Once all the moves are performed (i.e., the solution neighborhood have been explored) the best solution is selected and it becomes the new current solution *S*.

The process is repeated until there is no more improvement in solution quality. That is, no neighboring solution is better than the current solution.

CONCLUSIONS AND RECOMMENDATIONS

This paper introduces a new version of the VRP that considers a different and more environmental friendly objective function for the VRP. The new objective function minimizes Ton-Miles instead of total distance travelled by the vehicle. Consequently, the new version minimizes fuel consumption and CO_2 emissions. A mathematical formulation for the problem is presented. In addition, the problem is illustrated using a small problem instance. Also, a 2^{opt} LS is discussed. Finally, areas for future research for the GSVRP include generating a test dataset and developing efficient metaheuristics such as tabu search, simulated annealing or Ant Colonies.

References

- Dantzig, G.B. and Ramser, R.H. (1959) "The truck dispatching problem," *Management Science*, Vol. 6, pp pp. 80-91.
- [2] Garey, M. R., Johnson, D. S. (1979) "Computers and Intractability: A Guide tothe Theory of NP-Completeness," W. H. Freeman & Co., New York.
- [3] Marinakis, Y. and Migdalas, A. (2007) "Annotated bibliography in vehicle routing," *Operational Research*, Vol.7, No 1: pp. 27-46.
- [4] Parragh S. N., Doerner K. F., and . Hartl R. F. (2008) "A survey on pickup and delivery problems Part I: Transportation between customers and depot," *J Betriebswirtschaft*, Vol 58 pp,21-51.
- [5] Parragh S. N., Doerner K. F., and . Hartl R. F. (2008) "A survey on pickup and delivery problems. Part II: Transportation between pickup and delivery locations," *J Betriebswirtschaft*, Vol 58, pp.81-117.