USING CHAOS THEORY TO UNDERSTAND FIRM PERFORMANCE: A PHASE SPACE ANALYSIS

William "Rick" Crandall, Univ. of North Carolina at Pembroke, Pembroke, NC 28372 Richard E. Crandall, Appalachian State University, Boone, NC 28608 John A. Parnell, University of North Carolina at Pembroke, Pembroke, NC 28372

ABSTRACT

In the management literature, chaos theory has been used primarily as a metaphor to understand organizational phenomena. Using metaphors to understand organizations has gained much acceptance, thanks to the pioneering work of Morgan (1986). However, chaos theory's value as a metaphor is overused and offers little that cannot already be explained using existing theories and frameworks.

Because chaos theory is a mathematical theory, we believe its mathematical principles offer the greatest application to the management literature. We propose using one tool of chaos theory, phase space, to understand firm performance. We illustrate the use of a phase space analysis using data from a publicly held firm, Pizza Inn.

INTRODUCTION

The use of chaos theory in management research has been confined mainly to a metaphorical approach. In the management and organizational theory realms, the use of metaphor is well known, thanks to the pioneering work of Morgan (1986). However, none of Morgan's metaphors are based solely on a field of mathematics. In contrast, chaos theory is grounded in mathematical principles. Herein lies the problem; what are management scholars do to do with this theory? Two options can be identified. First, scholars can continue to apply chaos theory primarily as a metaphor. Second, the mathematical concepts inherent to chaos theory, particularly its use of phase space, can be more actively applied. Evidence suggests that the first option has been overused while the second option is currently underused.

In this paper, we address the unwarranted enthusiasm and often imprecise application of chaos theory as a metaphor, as well as its underuse as a formidable mathematical application for management research. The paper begins with a brief history of chaos theory. Next, we overview its main components and discuss its misapplication as a metaphor. We then discuss how one tool used in chaos theory, phase space, can be used to examine firm performance history. We conclude with implications for management researchers.

BACKGROUND

Although there are a number of important figures in the history of chaos theory, a key starting point would begin with Edward Lorenz. As a meteorologist, Lorenz was working with equations in a weather forecasting model he was developing involving data on temperature, air pressure, and wind direction (Briggs & Peat, 1989). At one point in his work, he decided to take a shortcut. Instead of inputting the values to six decimal places, he used three. The results he obtained were noticeably different from what he had anticipated; and that became the trigger event that led to a key component of chaos theory – sensitive dependence to initial conditions. Lorenz had stumbled on a discovery that indicated a small change in the initial conditions of a system could lead to very different outcomes as the system evolved. This later evolved into the famous butterfly effect. This effect states when a butterfly flaps its wings,

perhaps in some exotic part of the world like Hawaii, it can initiate a series of air currents that influence other weather events that eventually cause a hurricane in Florida. Of course, should the butterfly flap its wings in a different direction, the hurricane could develop somewhere else, perhaps Cuba or Mexico. In the butterfly illustration, we see an example of how a slight change in initial conditions can lead to a vastly different outcome in the life of the system under study.

The bestselling book by James Gleick (1987) made chaos theory more understandable to those outside the mathematical and physics disciplines. Soon, social scientists, psychologists, and even a few management scholars found an interest in chaos theory. To these researchers, chaos theory offered an interesting, nonlinear framework that could be used as a lens to understand the complex social and psychological interactions that comprise individual and organizational behavior.

Chaos theory also brought an abundance of interest from those who identified with the concept in nonmathematical ways. One viewpoint was that chaos carried with it a sense of mystery and excitement about life (Stoppard, 1995). The appeal of chaos theory has also been compared to a romantic appreciation of disorder that accompanies a corresponding reaction against the scientific appreciation for order and symmetry (Friedrich, 1988; Smith & Higgins, 2003). Polkinghorne (1993), a priest, articulated that chaos theory helps to describe the divine plan for the universe. While these viewpoints are interesting, they are inconsistent with the original intent of chaos theory. In fact, in one sense, chaos theory is not actually a theory at all, but an extension of nonlinear mathematics (Bolland, & Atherton, 1999).

One of the aspects of chaos theory that has also contributed to its increasing popularity is its visual dimension (Smith & Higgins, 2003). Attractors, a key component of chaos theory, can be graphed and many of these display an aesthetic appeal that likens it to computer art (Carey, 1995). Many of the articles that we reviewed on chaos theory depicted the famous Lorenz butterfly attractor. Add some color to this attractor and one can create an amazingly beautiful graphic. Certainly, this ability to take a seemingly difficult mathematical process and make it visually appealing adds to the mystique and popularity of chaos theory. In addition to the Lorenz butterfly, the Mandelbrot set has probably been the most famous visual artifact from chaos theory.

THE COMPONENTS OF CHAOS THEORY

Chaos is not a state of randomness or disorder, but rather a state whereby phenomena that appear to be unrelated actually follow an unknown or hidden pattern (Smith, 2002; Tetenbaum, 1998; van Staveren, 1999). This hidden pattern is called an attractor and it can be visually observed through the plotting of data in phase space. Furthermore, the relationships among the variables in a chaotic system are existent, but are "rather vague and at best, difficult to discern" (Kiel & Elliott, 1996:2).

Chaotic systems possess two characteristics, sensitive dependence to initial conditions and unpredictability in the long run. Each of these are discussed in more detail below.

Sensitive Dependence to Initial Conditions

Lorenz (1993) noted that a slight change in the initial input of his data led to vastly different results in his weather model. This now famous occurrence led to the popular "butterfly effect" illustration mentioned above. Lorenz also discusses another system in his book that is sensitive to initial conditions, the path of sleds descending down a snowy slope. In this example, he illustrates with diagrams, how seven sleds can end up in different areas stopping areas at the bottom of a hill, even though they may have started their descent within ten centimeters of each other. Of course, the paths the sleds take will change directions,

depending on the location of small humps or moguls along the route of its descent. This example also illustrates the concept of sensitive dependence to initial conditions, one of two general conditions that characterize a chaotic system.

Unpredictability in the Long-run

The behavior of a chaotic system cannot be predicted in the long-run. At best, there may be some accuracy in making short-term predictions. The weather is an example of a chaotic system that cannot be determined on a long-term basis, but can be predicted successfully in the short-run (Lorenz, 1993). For example, as one of the authors of this paper prepared an earlier version of this section, an ice storm was predicted later that evening. Local schools were on a two-hour delay the following day. While this impending ice storm was predicted with a reasonable amount of accuracy, nobody several months ago could have readily predicted such a storm would arrive on that particular evening.

These are the two main characteristics of a chaotic system, (1) sensitive dependence to initial conditions and (2) an inability to predict the final outcome of the system on a long-term basis. There are other components as well, but these first two characteristics represent a good starting point. Next, we describe bifurcations, attractors and feedback.

Bifurcations

A bifurcation is a point in the behavior of the system where the outcome can actually oscillate between two possible values in alternating time periods. The discovery of a bifurcation was made by Robert May, a biologist who was conducting a population model experiment (Gleick, 1987). As May increased the growth rate in his model, the population increased until it reached a critical point. At that point, the population would alternate values on a two year cycle, reaching a certain value the first year, followed by a lower value the next year, then returning to the original value the third year, and so on. May increased the growth rate again until a new critical point was reached, this time a period four or second bifurcation took place. At this new critical point, the population alternated in a four-year cycle. Increasing the growth again led to still more bifurcations until the model reached a state where almost any value was possible. At this point, the system was now "in chaos" because the population did not settle down to any predictable level. Hence, the lack of predictability is central to the notion of chaos.

May continued to increase the growth rate even while the system (i.e., the population level) was in its chaotic state. Surprisingly, when the growth rate reached a certain point, the system settled back down to a constant three-year cycle. In other words, it had moved from a state of chaos back to a state of order. However, as he increased the growth rate again, the system returned to chaos. In fact, the system continued to move in and out of chaos as the growth level was increased. Figure 1 illustrates a simplified bifurcation diagram, more correctly referred to as a logistics map (Guastello, 2008).

Figure 1 – A Simplified Bifurcation Diagram





X- Axis: System Parameter that is adjusted

(Robert May used the population growth rate as the system parameter).

Adapted from – Crandall, W.R., Parnell, J., & Spillan, J. (2009). *Crisis Management in the New Strategy Landscape*. Thousand Oaks, CA: Sage Publishing, p. 222.

Attractors

In chaos theory, an attractor is a pattern that forms when the behavior of the system is plotted in phase space (Lorenz, 1993). When the points are joined by a line in a chronological order, a pattern develops that can resemble a point, orbit, or some kind of unusual pattern. The unusual pattern is also referred to as a strange attractor.

Attractors range from being fairly simple to vastly complex. Four types of attractors have been identified: Point, pendulum, torus, and strange (Barton, 1994; Hudson, 2000; Stam, 2003). In phase space, a point attractor is depicted as a single plot on a graph. This is because the system behavior remains consistent over time. The pendulum attractor, also referred to as a period attractor, resembles an orbit when drawn in phase space. An example of a period two attractor appears later in this paper. The torus attractor is a more complex pattern that forms an orbit, but also contains points within the orbit, thus resembling a donut when graphed in phase space. Finally, the strange attractor, sometimes referred to as a fractal, is a complicated pattern that exists when the system is in chaos. The attractor is called strange because its shape may or may not resemble any known pattern.

Feedback

There are two types of feedback to consider, negative and positive. Negative feedback seeks to return a system to its original or normal state. For example, when one is speaking into a microphone, the speaker may find themselves adjusting their voice, either speaking softer or louder in order to find the right volume. Other evidence of negative feedback would be if the speaker moves the microphone closer or further away; or if the sound technician adjusts the volume on the sound board.

Positive feedback on the other hand amplifies the deviations in the system and pushes it further away from its original or normal state. Murphy (1996) notes a form of positive feedback that would occur if the speaker were to place the microphone near the loudspeaker. Such an event would cause the sound to distort and amplify itself throughout the system. The output from the speaker becomes input to the microphone, creating iterations which become louder and louder. Hence, positive feedback distorts and amplifies the deviations in the system.

In a chaotic system, positive feedback moves the system away from its original state to a new state. It is important to note that the process of iteration is necessary for the system to evolve into its new form. Hence, the use of time series data is necessary when the researcher is studying chaotic systems.

CHAOS THEORY AS A METAPHOR IN MANAGEMENT RESEARCH

Chaos theory was applied in the social sciences in the 1990s (Guastello, 2008). In the field of management research, its use has primarily been invoked as a metaphor. The practice of using metaphors to help explain the workings of an organization is not a new concept in managerial research. Morgan (1986) was a key player in generating enthusiasm for the use of metaphors to explain organizational behavior. However, Morgan did not use metaphors that were based on deep mathematics.

The dilemma of using chaos theory as a metaphor concerns the theory's intended use, to explain a unique mathematical state—chaos—where system behavior is neither orderly nor random. And herein lies an irony, social scientists and organizational scholars do not readily use elaborate concepts from chemistry or physics to explain social behavior, so why the strange attraction (pun intended) towards chaos theory? Indeed, the appeal of metaphors is a strong attracting force, one that can cause some misunderstandings, as we examine in the next section.

The Metaphor Problem

Misunderstandings concerning chaos theory stem primarily from its overuse as a metaphor, not in its mathematical use, since its mathematical use is limited in the management literature. While metaphors are useful to understanding complex systems, there is a tendency to extend them beyond their usefulness. Eigenauer (1993) pointed out that some scholars have used chaos theory to the extreme. In commenting on Gleick's book, he notes:

While Gleick's work is solid, it has led some to be captivated by chaos theory's fecund metaphorical terminology and elegant computer aided graphical images. Although those images ... show striking instances of order hidden within chaotic systems, too often they are used to forward the thesis that there are other systems, ranging from modern literary theory to stock market fluctuations, that also house deep structure amid their apparent disorder. The result is, on occasion, analysis that is based only on metaphor. Eigenauer (1993: 455)

Smith (2002:523) also concurred with a similar thought, "... some disciplines have already displayed a tendency to rely too heavily on purely conceptual applications of chaos theory. This is in danger of reducing chaos theory to a collection of metaphors, or worse still reducing it to just semantic innovation if the application is trivial."

One problem with using chaos theory as a metaphor is that it offers little that cannot already be explained using already existing theories or frameworks. For example, Bright and Pryor (2005) compare the four types of attractors to career decisions. While their article is interesting and does an excellent job of describing the various career dilemmas that we may face; their discussion of attractors does not add anything to our knowledge of careers. Put differently, the same article, without the discussion on attractors would have been sufficient. In another example, Sellnow, Seeger, & Ulmer (2002) offer an excellent assessment of the 1997 Red River flood in Minnesota and North Dakota. However, their attempts to tie in chaos theory to the discussion add little to our knowledge and even distracts from the central point; their analogy of the strange attractor to the support agencies involved in managing the flood is interesting, but not necessary.

While metaphors are useful in gaining new insights, its overuse can lead to problems. Most metaphors begin to break down in their usefulness when overused because exact parallels between the metaphor and the phenomena under study are inappropriate (Barton, 1994, Chubb, 1990). While chaos theory can enable us to think about our research question in a different perspective, we must take care not to overextend the metaphor.

Misunderstandings of the Meaning of Chaos and its Accompanying Components

Psychology was one of the first social sciences to embrace chaos theory as a viable tool of analysis. However, as Barton (1994) notes, some misunderstandings of chaos have emerged. He cites Bütz's (1992) use of chaos as "overwhelming anxiety," a state that bears little resemblance to the mathematical state of chaos described in the original theory. In their critique, Kincanon and Powel (1995) cite the example of Sappington (1990), who stated that chaotic systems are unpredictable. On this point, one should recall that chaotic systems are difficult to discern but not completely unpredictable, especially when graphed in phase space.

Using chaos theory as a metaphor has also led to the inappropriate use of the components of the theory. For example, Bright and Pryor (2005) attempted to link attractors to careers. They discuss how point attractors can be likened to a particular vocational goal such as being promoted to the next level in the corporation. While this analogy is interesting, it is not even remotely possible to call this a point attractor, even as a metaphor. Likewise, the strange attractor has been likened to a number of items including the corporation's value system (Frederick, 1998).

EXAMINING FIRM PERFORMANCE THROUGH PHASE SPACE

Chaos theory is a mathematical theory and therefore, its use should include mathematics (Elliott & Kiel, 1996; Faber & Koppelaar, 1994; Smith, 1995; Smith, 2002). In this section, we reflect on the future of chaos theory in management research by offering an application of phase space analysis, a mathematical tool of chaos theory.

The first requirement for such an analysis is time series data, as it is the primary domain area for studying chaotic behavior (Haynes, Blaine, & Meyer, 1995; Hudson, 2000). This requirement is necessary given that the key to understanding sensitive dependence to initial conditions is to acknowledge the iterative process that occurs in the system over time. Iterations increase the magnitude of the deviations (positive

feedback) over time, causing the final outcome of the system to be much different than its starting point. It is the impact of positive feedback that moves the system away from its original starting point.

Choosing Variables to Study

As management scholars, we typically seek to identify the impact of select independent variables on one or more dependent variables. We often measure the strength of these variables as well as their interrelationships, first through correlation analysis (which also helps check for multicollinearity and other concerns), and then multiple regression analysis (which helps determine the strength of the independent variables and the model's overall usefulness, via the R-square). However, in chaos research, we seek to identify the pattern of movement of the system under study through time by graphing the system variables in phase space.

In this paper, we graph the two variables of revenue and profits. These variables were selected based on the following criteria:

- Both variables can be captured in time series data.
- Revenue and profits are absolutely essential for the long-term sustainability of the firm.
- Both variables function as a proxy for how the firm is operating.
- Conceptually, both variables are easy for students to understand and aid in viewing performance from a different perspective.
- There is a limit to the number of variables that can be analyzed in phase space. Although any variables of interest to the researcher can be graphed in phase space as long as they can be depicted by time series data, the two applied herein are of key concern to both scholars and practitioners.

Graphing Variables in Phase Space

Phase space is used in the physical sciences, but may not be very familiar to management researchers. In phase space, the properties of the system under study are plotted at a point in time. With each iteration, another plot is made, which eventually results in a pattern (i.e., an attractor) when the plots are joined in chronological order by a line. "Diagramming the movement of a system's variables in phase space reveals the curious byways of an otherwise hidden reality" (Briggs & Peat, 1989: 32). Put another way, the pattern of a time series that looks haphazard may actually have a hidden structure to it if we look at it in a different manner, through phase space.

Phase space can be graphed using variables that the researcher desires to study. With one variable, phase space is typically graphed by placing the current data point from a time series on the y-axis and the prior data point from the previous period on the x-axis. This one variable arrangement is also referred to as pseudo phase space (Williams, 1997). Stam (2003) studied the single variable, EEG patterns (brainwaves), and plotted attractors in an attempt to identify conditions that can lead to an epileptic seizure. In the production operations management literature, Giannelos and associates (2007) used the single variable, flow time, in assessing dispatching policies for manufacturing jobs.

Plotting with two variables is also possible in phase space. For example, mechanical systems have been examined in phase space using position and velocity while ecological systems have been studied in terms of the population size of the species being studied (Briggs & Peat, 1989). In medical research, Reidbord and Redington (1992) constructed a phase space with heart rate and the patient's behavior state as the study variables. In the area of public administration research, Kiel (1993) constructed an attractor in phase space using time series data involving labor costs associated with service requests. In the area of strategic

management, Priesmeyer & Baik (1989) examined revenue and profit changes among retailers and identified attractors in phase space.

The Priesmeyer & Baik (1989) work sets the stage for the following discussion. In the next section, we assess the movement of revenue and income variables in phase space. We then offer a glimpse into how one firm exhibited a strange attractor in phase space.

UNDERSTANDING PHASE SPACE

Thus far, we have suggested that a research study using a chaos theory approach needs both time series data and a means of graphing the data points in phase space. In this example, we examine total sales and net income as they appear in phase space. In phase space, we are capturing the movement of the system variables through time. To accomplish this, we adjust our two study variables to reflect this requirement. Thus, we need to capture the variables as the "change in total sales", which will be shown on the x-axis, and the "change in net income", which will be depicted on the y-axis.

To obtain the change in total sales (x-axis coordinate), the difference between the present total sales for the fiscal quarter and the total sales for the previous quarter are calculated. The same procedure is used to calculate the change in net income (y-axis coordinate), using the net income (loss) figures.

Change in Total Sales = Total Sales
$$_{\text{current}}$$
 – Total Sales $_{\text{previous}}$ (1)

Change in Net Income (loss) = Net Income
$$_{current}$$
 – Net Income $_{previous}$ (2)

We begin with a graph (Figure 2) that depicts the two study variables, change in total sales (X-axis) and change in net income (Y-axis). Note that the upper right quadrant would be the most desirable for the firm, as it indicates consecutive periods of increasing total sales and net income.



Suppose that a firm was able to increase sales for three fiscal periods by exactly \$400. Suppose also that for each period, the firm increased net income by exactly \$200. If this oversimplified situation were graphed in phase space, it would be plotted as one single point. This would be an example of a point attractor as shown in Figure 3.



Now, suppose that in the next fiscal quarter, the firm experienced a decrease in sales by \$400 and a decrease in net income of \$200. The resulting plots would look like the ones in Figure 4. When the two points are joined by an arrow, the line slopes from the top right to the bottom left. It is important to remember that plots in the lower left or right quadrants do not necessarily mean the firm experienced a net loss, only that it experienced a <u>decrease</u> in net income for that fiscal period.



Assume that in the next fiscal quarter, the firm experienced an increase in total sales of \$200 and an increase in net income of \$200. Remember, these are <u>changes</u> from the <u>previous</u> fiscal period. This is not the same as saying that the firm's sales and net income were \$200, a situation that would be impossible to achieve. Figure 5 illustrates this change, which is a return to the original quadrant.



Assume also that in the subsequent quarter the firm experienced a decrease in total sales of \$175 and a decrease in net income of \$200. When the plots are joined together by the arrows, one can see the beginnings of a pendulum or more accurately, a period 2 attractor as shown in Figure 6.



When the remaining points in this example are graphed, what results is an obvious pattern, an attractor that cycles between the two quadrants. In fact, this pattern could be quite normal for any business that experiences seasonal business cycles where sales and net income fluctuate from one period to the next. One of the characteristics of the period 2 attractor is that it is easy to determine which quadrant the next plot will fall in. In addition, we propose this particular attractor is more the norm for a firm that is operating well. The completed example of the period two attractor appears in Figure 7. The dataset used to generate this hypothetical example is in Table 1.



 Table 1 - Data Set for the Above Example

Fiscal	Change in Total	Change in Net
Period	Sales	Income
1	400	200
2	-400	-200
3	200	200
4	-175	-200
5	250	150
6	-300	-250
7	250	250
8	-300	-200

Consider another example of a period two attractor using real data. Home Depot exhibits a consistent two phase oscillation from the upper right to the lower left quadrants; an indication that sales and net profits are moving in a normal cyclical pattern. Figure 8 illustrates Home Depot's performance in phase space.



Data Source – Mergent Online

Figure 9 depicts the same Home Depot data graphed in the traditional manner. Total sales are depicted by the top line while net income is shown by the bottom line. This graph has two advantages over the phase space graph depicted in figure 8. First, it shows that net income does not result in a loss during any fiscal period, a feature that is not available in phase space. Second, the graph is more visually appealing vis-àvis the scale of the total sales relative to net income.

On the other hand, the phase space graph has two advantages over the traditional graph. First, each plot represent the state of the two variables together, total sales and net income. The plot is a representation of the state of the organization at a particular time (or phase). Second, the graph shows more sensitivity to changes in the system behavior, as evidenced by the shape of the attractor.



Figure 9 – Home Depot Data in Traditional Time Series Format

Data Source – Mergent Online

We have thus far presented three examples of phase space attractors; one example of a point attractor, one hypothetical period two attractor, and one real world period two attractor using time series data from Home Depot. However, none of these examples exhibits a state of chaos, an assertion we base on three factors. First, recall the two conditions of chaotic systems, (1) sensitive dependence to initial conditions, and 2) unpredictability in the long-run. The system does not appear to be sensitive to initial conditions. All three examples violate both conditions. First, the point attractor does not deviate at all from its initial conditions; it remains at its designated point in phase space. Second, because the point is stable, it is easy to predict on the long-run, hence, no chaos. Both period two attractor examples are also not sensitive to initial conditions, as their consistent orbits bring the system back into a predictable pattern, which also disproves the second condition, that it is unpredictable in the long-run, hence, no chaos.

ASSESSING PIZZA INN

Based on previous knowledge, we suspected the restaurant chain Pizza Inn as a prospective example of chaos. Specifically, we suspected that the company would depict a chaotic attractor when its performance variables of sales and income (loss) were graphed in phase space.

The reasoning was that a poor performing firm would be more difficult to predict in terms of performance from one fiscal quarter to the next. Hence, its phase space history would be chaotic. The result would be a strange attractor that did not appear to follow a point or period two cycle. This methodology is consistent with Priesmeyer & Baik's (1989) assessment of retail firms.

To test our proposition, we collected time series data from Mergent Online for Pizza Inn Ten years of sales and income data was retrieved. Next, we continue our explanation of the results.

Results

Figure 10 depicts thirty fiscal periods for Pizza Inn using the traditional format for time series data. Total sales are shown by the line on the top and net income by the line on the bottom. The general trend has been a decrease in sales over the 10 year period. Profits have been uneven with a brief spike at the end of 12/29/2002 and a loss in 9/24/2006.



Figure 11 depicts the data in phase space. It is immediately evident that the data does not follow a period two attractor as illustrated earlier. In fact, the plots are represented in each of the four quadrants.



The shape itself is unusual, as this data depicts what chaos theorists would call a "strange" attractor, an indication of chaos. However, in order to understand this firm performance better, we need to assess it in smaller sections. In the discussion that follows, we reexamine the graph in two-year increments to gain a better understanding of how this system evolved. Figure 12 begins our analysis as it covers six fiscal periods from 3/25/2001 to 12/28/2003.



The cycle begins at the start point which is the end of the 12/24/2000 period. The first period ends with a slight decrease in sales, with a slight increase in income, putting the plot in the upper left quadrant. This in itself is somewhat of an unsual situation, as revenue declines are not typically associated with profit increases. At the end of the second period, sales have increased and income has decreased, again, an unusual situation. The third period ends with a decrease in both sales and income, a situation which intuitively, is easier to comprehed. The fourth period also realizes a decrease in both sales and income. By the end of the sixth period, another unusual occurrence has occurred, an increase in income despite a small decrease in sales.

It is difficult to determine from this assessment where the next plot will take place because no pattern has emerged. Considering figure 13, note how the start point is the same as the end point of the previous graph. What is immediately noticeable is that this cyle is very different from the previous two year cycle.



At the end of the first period, both sales and income have taken a substantial decline as the plot moves to the lower left quadrant. The second period sees a major increase in sales, with a slight increase in income. The third period ends with a sales decrease with a slight increase in income. From there, Pizza Inn remains in this quadrant with a continued decrease in sales, with a small increase in income by the end of the fourth period. The next two quarters end with both a decrease in sales and income. Note that firm sustainability cannot be maintained in this lower left quadrant over an extended period of time. Of the six points plotted in this phase space, three are in this lower left quadrant. Only one point land in the upper right quadrant, an indication of healthy firm performance. This could be a cause for concern if Pizza Inn's performance continues to land in this quadrant.

Figure 14 depicts the next two-year graph which appears at first to be a period two attractor. This is not the case, however, as is evident from the location of the points in the quadrants. A period two attractor would have an equal number of points in two of the four quadrants. In this example, four points fall in the lower left quadrant, two are in the upper right, and one point is in the lower left quadrant.



Again, the start point is where the end point was located in the previous graph. At the end of the first period, there is a decrease in sales with a slight decrease in income. By the end of the second period, sales and income have decreased in a greater amount. A rebound in sales occurs by the end of the third quarter, but income is still continues to decrease. Note that this indicates three consecutive quarters of income decrease, a situation that is not sustainable in the long-run. However, by the end of the fourth quarter, there has been a slight increase in both sales and income, certainly some good news. However, by the end of the fifth period, there is substantial drop in both sales and income. What is interesting next is the large increase in sales and income by the end of the sixth period. This one was certainly difficult to predict, given the haphazard performance from the previous five periods.

Figure 15, the next two-year graph, depicts a pattern that is different from the previous ones. That this pattern is different is another indication that Pizza Inn's performance over this ten year period is most likely an example of chaos.



The first period ends with a decrease in sales and an increase in income. Period two ends just the opposite with an increase in sales with a slight decrease in income. The third period rebounds with an increase in sales and income followed by a decrease in sales with a small increase in income for the fourth period. Once again, it is difficult to predict exactly where the next point will fall. The fifth and sixth periods experience decreases in sales and income.

Figure 16, the last two-year period graph, is the most unusual one yet. The pattern appears like a figure eight, with points falling in all four quaderants. Still, it is difficult to determine where the pattern will evolve in the years that will follow.



The first and second periods both depict a decrease in sales and a decrease in income, although the income decrease in period two is relatively small. The third period realizes an increase in sales with a slight increase in income. This is followed by a decrease in sales and income in the fourth period. The fifth period sees a major increase in sales coupled with a significant decrease in income. The sixth period ends with an unusual decrease in sales accompanied by an increase in income.

DISCUSSION

In the previous section, we illustrated how a tool of chaos theory, phase space, can be used to track the firm performance variables of sales and income. We identified a strange attractor in the ten year operating period of Pizza Inn. We then examined the evolution of the system in two-year increments. Each graph displayed a different pattern (attractor) from the previous graph. It was difficult, if not impossible to determine the quadrant in which the next fiscal period would fall and the pattern the system will take in the future. We conclude that this time series data displays the characteristics of chaos.

This conclusion meets the criteria of chaos. Recall that there are two overarching conditions that must be met, (1) sensitive dependence to initial conditions, and (2) unpredictability in the long-run. Proving the first condition is not possible, as it would necessitate the assessment of two almost completely identical pizza chains. Only a few differences between the two chains would be permissible, such as different managers and employees. Then, over a period of time, the chains would eventually diverge and evolve into very different entities. It is the iterations that would make the chains different, and hence, sensitive to their initial conditions. Indeed, only in a computer simulation can such a scenario be evaluated.

As a proxy, we could consider case studies of actual firms. Firms are constantly being formed, but only a small percentage will survive for a substantial period of time. Likewise, in the retail pizza

industry, many pizzerias have come and gone. Virtually every pizza chain started with only one restaurant, some growing large (e.g., Pizza Hut), and some not so large (e.g., Pizza Inn). Many pizzerias have remained small, and most to this day are single proprietorships with only one unit. And yet, they all still began the same way, with an idea, some capital, and a single store. The sensitive dependence to initial conditions comes into play when we see how the hundreds, and indeed thousands of pizza restaurants, have evolved into different forms. Perhaps sensitive dependence on initial conditions is the modus operandi in the business world. Pizza Inn then, started like the other restaurants did, but emerged differently from Pizza Hut, California Pizza Kitchen, or CiCi's. Indeed, all of these companies displayed sensitive dependence to their initial conditions.

As for the second condition, unpredictability in the long-run, our data does support this conclusion with the Pizza Inn chain. The haphazard or strange shape of the attractor displays a pattern of firm movement in phase space that is difficult to determine. It is impossible to predict what Pizza Inn will look like next year. Indeed, we cannot even predict what its attractor in phase space will look like over the next year. Compare this with the period two attractor of Home Depot. With this company, we can say, with some degree of confidence, that its phase space cycle is likely to continue its pattern.

WHAT DOES A CHAOTIC SYSTEM TELL US?

We thus conclude that Pizza Inn displays a chaotic performance with regard to its sales and income. But what benefit exists for understanding that a time series is chaotic? We propose the following three reasons.

1. A chaotic system is difficult to predict.

Each of the two-year graphs illustrates the difficulty in predicting just where the system will move next. Because phase space is sensitive to changes in the system, we can see that firm performance is haphazard in terms of which quadrant it will land in next. This pattern is not apparent in a normal time series data graph; but reveals irregularities when the same data is graphed in phase space.

2. A chaotic system may reveal that something unusual is occurring.

The Pizza Inn example illustrates a number of point plots that fall in the upper left hand quadrant, indicating a decrease in sales and an increase in income. Scholars may wish to investigate the causes of this anomaly. While not necessarily dysfunctional, this situation is out of the norm, as one would expect income and revenues to be positively correlated. A look at the time series graph that plots sales and income (as opposed to change in sales and income, as we see in phase space) does not readily reveal this particularity. However, when we venture into phase space, the unusual is revealed more clearly.

3. A chaotic system can serve as an early warning indicator.

Traditional time series data, as illustrated in figures 9 and 10, can indicate when there is a performance problem. In fact, figure 10 clearly shows a closing gap between sales and profits. However, what we can glean from this figure is limited. Because of its sensitivity to shocks, phase space can help us see early on if a problem is emerging in the early stages in a way that traditional time series cannot. Figure 17 offers a visual summary of the four quadrants in phase space.



Figure 17 – A Summary of the Four Quadrants in Phase Space

The growth and full decline quadrants are intuitive. Over the long term, income tends to rise and fall with revenue. Strategic managers at most firms seek a position in the growth quadrant. They attempt to grow their businesses by increasing sales, and ultimately profits. Remaining in this quadrant could be viewed as inherently desirable for most firms.

In contrast, the full decline quadrant is the least desirable position. Income steadily declines, ostensibly due to decreases in revenues. Firms in this quadrant might seek to increase income by cutting costs, but this can be counterproductive if doing so negatively impacts product quality, service, or other factors that drive revenues.

Executives at firms consistently occupying either the growth or the full decline quadrants gain little by applying phase space. The general health or lack thereof is typically clear. Strategic managers whose firms frequently occupy the remaining two quadrants—unusual and partial decline—can glean more from this type of analysis.

The unusual quadrant is counterintuitive; firms occupying this space enjoy an increase in income while revenues are declining. This occurrence could be a short term phenomenon that results from aggressive cost cutting. The relationship between revenue and income is a critical one for such firms as their managers must struggle to discern between effective and ineffective approaches to reducing expenses. For example, a reduction in the advertising budget might be appropriate if promotional efforts do not result in sufficient increases in revenue, but inappropriate if sales declines precipitously as a result. Distinguishing between these two possibilities is a complex challenge, but one that executives must traverse.

The partial decline quadrant is intriguing. Executives at firms in this quadrant face problems similar to those whose organizations occupy the unusual quadrant, but from a different angle. In the partial decline quadrant, increases in revenues are not translating into profits. A number of culprits could explain this phenomenon, from ineffective cost controls to an increase in the intensity of competition, requiring price reductions. Again, the key here is for strategic managers to identify the cause(s) so that remedial action can be taken.

On the surface, examining each quadrant independently is not a difficult exercise. The complexity emanates from the *movement* of the organization across quadrants over time. Identifying these shifts in a visual manner is perhaps the greatest contribution of phase space, while interpreting their collective meaning is the greatest challenge to decision makers.

ADVANTAGES, LIMITATIONS, AND FUTURE DIRECTIONS

In this final section, we offer advantages, limitations, and future directions for research.

Conclusions and Limitations

This study applies a tool from chaos theory, phase space, to help us better understand time series data that involves strategic variables. As a metaphor, chaos theory is overused and appears to offer little in terms of distinctive descriptive or prescriptive value. We emphasize the mathematical properties on which chaos theory was founded, albeit on a limited basis, to help us understand firm behavior. This paper demonstrates that phase space offers unique advantages that are not available with traditional time series data.

Phase space is not a technique to replace any of the more traditional approaches to time series analysis; instead, it can supplement our understanding of firm performance. Hence, the use of chaos theory is not a superior approach to analysis, as some have suggested, but instead, another tool in our box of techniques to aid the management researcher.

The primary limitation of this study was that only one company was analyzed. As we explain next in future directions, studies involving multiple companies are desirable. Each organization charts it own course through phase space. Similar patterns attributable to similar causes may suggest a common organizational response. Outlining such prescriptions can be invaluable, but the present study is only the beginning.

Future Directions

Several research opportunities exist. A potential research stream would be to conduct similar phase space analyses on other firms. Comparisons of companies in the same industry may reveal similarities in terms of how the attractors that are graphed. Put differently, are there industry specific attractors? We also propose that the following questions be addressed in future research endeavors.

- What conditions would cause sales to decrease and income to increase (upper left quadrant)? An occasional drift into this section would be understandable, particularly if the numbers are low. But a high decrease in sales with an increase in income would be interesting to pursue. What would cause this unusual event to happen?
- Are there certain shapes of attractors that are sustainable in the long-run? We identified the period two attractor that oscillates between the top right and bottom left quadrants as being normal for a company experiencing seasonal influences. However, there may be other more

complicated attractors that exist that are also sustainable in the long-run. For example, a period three or period four attractor may be possible.

- **Do certain shapes of attractors precede episodes of chaos?** Phase space analysis may reveal attractors that act as precursors to chaotic episodes. If so, what would these preliminary attractors look like? Answering this question would assist practitioners in identifying problematic patterns in their organizations.
- Can other variables be graphed that would add to our understanding of the organization's performance? For example, what if research and development (R&D) expenditures were graphed in phase space with net income or net sales as accompanying variables? Would the resulting attractors yield meaningful information? R&D is mentioned because of its use as a means of differentiation. R&D is particularly characteristic of the prospector business strategy and the differentiation strategy (Parnell, 2008).
- What causes a company to enter into chaos or emerge from it? Recall that chaos is not necessarily a time when the company is experiencing a net loss. Instead, chaos occurs when the firm is experiencing changes in sales and profits that are difficult to predict from one fiscal period to the next. We are not referring to the inability to predict the amount of change, but the *direction* of that change (i.e., whether it is increasing or decreasing).
- How can the use of chaos theory be blended with conventional forecasting methods? One of the most sensitive areas for public companies is in forecasting future expectations of revenue and income growth, or decline. Investors would no doubt have more confidence with companies operating with a period two attractor than one operating in a chaotic state.

Given this definition of chaos, the question now becomes, what causes a company to enter periods where it is difficult to predict the direction of sales and the direction of profits (again, either increasing or decreasing)? Likewise, what conditions enable a company to depart from these cycles of unpredictability (chaos)?

The application of chaos theory to the strategic management field remains largely undeveloped. This paper can serve as a catalyst for developing this line of research, in both descriptive and prescriptive terms.

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