

The Green Vehicle Routing Problem

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ABSTRACT

This work introduces the Green Vehicle Routing Problem (GVRP). The GVRP is an extension of the well-known vehicle routing problem (VRP). Moreover, the GVRP includes an objective function that minimizes weighted distance. Minimizing weighted distance reduces fuel consumption and consequently CO₂ emissions. Therefore, the GVRP is more environmentally friendly than traditional versions of the VRP. This work presents a Mixed Integer Linear Program formulation for the problem and a Local Search algorithm to find local optima. Also, the problem is illustrated using a small problem instance.

1. INTRODUCTION

The Vehicle Routing Problem (VRP) is the designing of vehicle routes such that total distance traveled by all vehicles is minimized. The VRP has multiple applications in the fields of logistics and transportation. In fact, any company managing a fleet of vehicles that visits a set of customers needs to solve a VRP. The VRP has different objectives according to the goal to be accomplished. The most common objective functions (OFs) of the VRP are to minimize total distance traveled by the vehicles, and to minimize total traveled time. Often, traveled time is correlated with traveled distance; making minimizing total distance traveled the most common OF for the VRP. The VRP was introduced by [1] and was proved NP-Hard by [2]. In fact the VRP combines two difficult combinatorial optimization problems, the Bin Packing Problem (BPP) and the Travelling Salesman Problem (TSP). The BPP assigns cargo to vehicles, and the TSP optimizes vehicle routes.

As it was mention above, VRP problems are at the core of companies such as UPS and FedEx. There are multiple variations of the VRP according to problem OFs and assumptions. The most popular one is the Capacitated VRP (CVRP). In the CVRP vehicles either deliver or pick-up cargo (but not both) to or from a set of customers. Moreover, in the CVRP, customers are visited only once during the planning horizon. Another popular version is the VRP with Time Windows (VRPTW). The VRPTW allows customer to be visited only during certain periods of time. A third popular version is the pick-up and delivery VRP in which vehicles can visit customers more than once to either, deliver cargo, pick-up cargo or both. Finally, detailed reviews of VRP and its most important variations are available in [3], [4], and [5].

Previously, it was mentioned that the most common OF for VRP is total distance traveled minimization. Minimizing total distance traveled speed up time deliveries. However, routes that minimize total distance traveled are not the most efficient ones from the point of view of fuel consumption and CO₂ emissions. Therefore, this research proposes an alternative and more environmentally friendly version of the VRP that minimizes total weighted distance. The proposed problem is called the Green Vehicle Routing

Problem (GVRP). Moreover, weighted distance is defined by the product of vehicle weight and distance traveled (i.e., ton-miles or ton-km). In fact, fuel consumption is a function weighted distance. That is, a heavier truck uses more fuel than a lighter one when travelling the same route. Therefore, minimizing weighted distance minimizes fuel consumption, and consequently minimizes CO₂ emissions. A greener version for the TSP called the Green Single Vehicle Routing Problem (GSVRP) is discussed in [6]. Notice that a VRP with only one vehicle is reduced to the TSP.

The GVRP is computationally intractable since it is an extension of the VRP. Consequently, approximation algorithms such as tabu search and ant colonies are required to find good solutions in acceptable computational times. The remaining of this paper is as follows: section 2 formally introduces the GVRP and presents a Mixed Integer Linear Program (MILP) formulation; section 3 illustrates the problem using a small problem instance; section 4 provides a short discussion of GVRP difficulty; section 5 presents a Local Search algorithm (LS) for the problem; and finally, section 6 is conclusions and recommendations for future research.

2. PROBLEM DESCRIPTION

Since the GVRP is an extension of the VRP, the mathematical formulation for the GVRP is an expansion of the CVRP one. Consequently, a mathematical formulation for the CVRP is presented first, and then a Mixed Integer Linear Program (MILP) for GVRP is introduced. The CVRP is formally defined as follows: given a set of customers with given demands and a set of vehicles with limited capacities used to deliver demanded goods to customers, the CVRP consist of minimizing total distance traveled by all vehicles such that every customer is visited once. Following is a MILP for CVRP.

Indexes:

i, j Locations: $i, j = 1, \dots, L$; where 1 represents depot, and L is the total number of locations visited

Parameters:

d_{ij} Distance between locations i and j

q_i Customer i demand in weight units

Q Vehicle weight capacity

Variables

x_{ij} 1 if a vehicle visits location j immediately after serving location i
0 otherwise

u_i Arbitrary real variable

Objective function:

$$\min \sum_{i=1}^L \sum_{j=1}^L d_{ij} x_{ij} \quad (1)$$

Subject to:

$$\sum_{j=2}^L x_{1j} = K \quad (2)$$

$$\sum_{i=2}^L x_{i1} = K \quad (3)$$

$$\sum_{i=1}^L x_{ij} = 1 \quad j = 2, \dots, L; i \neq j \quad (4)$$

$$\sum_{j=1}^L x_{ij} = 1 \quad i = 2, \dots, L; j \neq i \quad (5)$$

$$Q - \left(Q - \max_{j \neq i} \{ q_j \} - q_i \right) x_{1i} - \sum_{j=2}^L q_j x_{ij} \geq u_i \quad i = 2, \dots, L \quad (6)$$

$$q_i \leq u_i \leq Q \quad (7)$$

$$x_{ij} \in [0,1] \quad (8)$$

$$u_i \geq 0 \quad (9)$$

Objective function (1) minimizes total distance traveled by all vehicles. Constraint set (2) ensures that K different routes start at the depot (i.e., one per vehicle). Constraint set (3) makes sure that exactly K routes arrive to the depot. Constraint set (4) guarantees that there is exactly one vehicle arrival to each location. Similarly, constraint set (5) guaranteed that there is exactly one vehicle departure from each location. Constrains sets (6) and (7) together ensure that vehicle capacities are not exceeded, and that vehicles routes include the depot location. Moreover, constraint set (6) is an extension of the Miller–Tucker–Zemlin sub-tour elimination constraint presented by [7]. Finally, constraint sets (8) and (9) define the nature of binary and continuous variables.

The GVRP can be defined as follows: Given a set of customers and a set of vehicles with limited capacities, the GVRP is to find the set of routes (i.e., one per vehicle) that minimizes total weighted miles while each customer is visited only once. The mathematical formulation for the GVRP is easily constructed by expanding the CVRP MILP presented above. That is, by adding a new variable set, a new parameter, changing the objective function, and adding three new constraints. These steps are showed below:

Additional parameter:

CW Vehicle Curb Weight (i.e., vehicle weight when empty)

Additional variable:

y_{ij} Total weight (CW + cargo weight) of vehicle traveling between locations i and j

New objective function:

$$\min \sum_{i=1}^L \sum_{j=1}^L d_{ij} y_{ij} \quad (10)$$

New objective function (10) minimizes total weighted distance (i.e., ton-miles, ton-km, etc).

Additional Constraints:

$$y_{i1} - CWx_{i1} \geq 0 \quad \forall i \quad (11)$$

$$(CW + Q)x_{ij} - y_{ij} \geq 0 \quad \forall i, j = 2, \dots, L \quad (12)$$

$$\sum_{i=1}^L y_{ij} - \sum_{i=1}^L y_{ji} = q_j \quad j = 2, \dots, L \quad (13)$$

$$y_{ij} \geq 0 \quad \forall i, j \quad (14)$$

Constraint set (11) ensures that vehicles arrive empty to the depot. That is, the weight of any vehicle arriving to the depot is CW . Constraint set (12) links route variables x_{ij} with vehicle weight variables y_{ij} . Constraint set (13) is a typical flow conservation constraint. Finally, constraint set (14) defines the nature of the additional variables.

3. PROBLEM INSTANCE

The GVRP will be illustrated using a small problem instance. The problem instance considers 10 customers that will be served from a single depot. In addition, the instance considers the usage of two vehicles, each with a capacity of 12 tons, and a curb weight of 8 tons. Table 1 includes depot and customer locations coordinates, customer demands in tons, and distances between locations in miles (distances between locations were round to integer values to facilitate the discussion). Figure 1 shows a chart with all locations. Notice that location 1 is the depot and it is represented by larger triangle in the chart. Also, numbers in parenthesis represent customer demands. For example customer at location 7 is expecting a delivery that weights 3 tons.

TABLE 1: PROBLEM INSTANCE DETAILS

L	Coordinates		q_j	d_{ij}										
	x	y		1	2	3	4	5	6	7	8	9	10	11
1	15	6	0	0	84	75	90	23	59	57	42	33	100	74
2	85	54	2	84	0	50	43	61	61	39	43	61	23	81
3	90	4	2	75	50	0	89	54	89	66	42	72	73	111
4	56	87	3	90	43	89	0	74	38	34	63	57	31	48
5	37	15	1	23	61	54	74	0	51	40	18	25	79	71
6	24	65	2	59	61	89	38	51	0	24	50	27	63	21
7	46	54	3	57	39	66	34	40	24	0	30	25	46	46
8	53	24	3	42	43	42	63	18	50	30	0	30	62	71
9	26	38	2	33	61	72	57	25	27	25	30	0	71	45
10	86	77	4	100	23	73	31	79	63	46	62	71	0	78
11	8	80	1	74	81	111	48	71	21	46	71	45	78	0

FIGURE 1: GRID WITH LOCATIONS

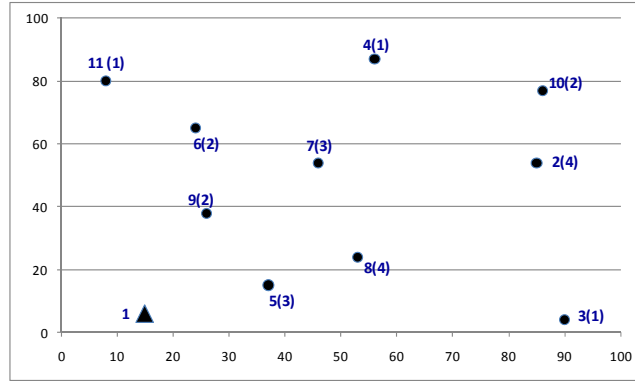
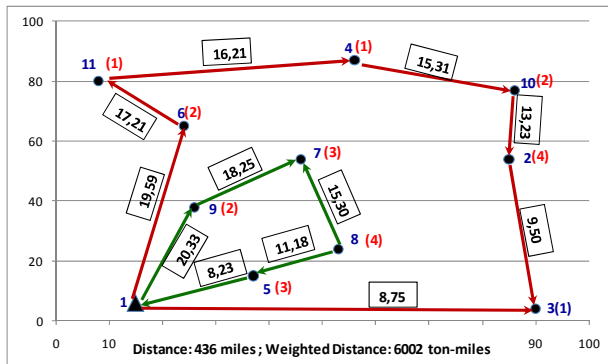
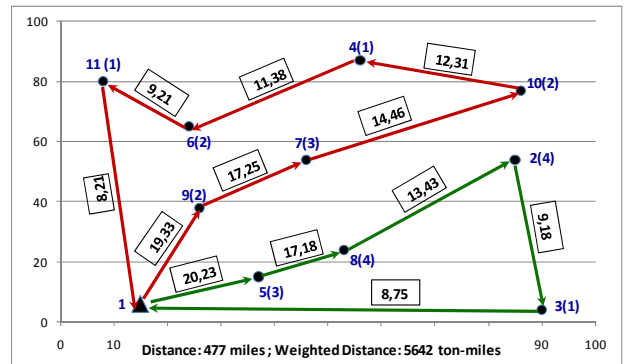


Figure 2(a) shows the optimal solution for the VRP version of the instance. The VRP optimal solution was obtained using the MILP described by equations (1) through (9). Moreover the model was coded using OPL and solved with the academic version of IBM ILOG CPLEX Optimization Studio 12.2. For the VRP optimal solution, the total distance traveled by both vehicles is 436 miles. Arrows indicate vehicle directions, and boxes next to arrows show miles traveled between locations followed by vehicle weights. Total weighted distance for this solution is computed by adding the products of all pairs in the boxes. In fact, weighted distance for this solution is 6,002 ton-miles. Similarly, the optimal solution for the GVRP was obtained by solving the MILP described by equations (2) though (14). As before, the MILP was coded using OPL and solved with the academic version of IBM ILOG CPLEX Optimization Studio 12.2. Figure 2(b) shows the optimal solution for the GVRP. Notice that the optimal solution has a weighted distance value of 5,642 ton-miles. Also, in the optimal solution for the GVRP, the vehicles traveled a total of 477 miles.

FIGURE 2: VRP AND GVRP OPTIMAL SOLUTIONS



(a): VRP Optimal solution



(b): GVRP Optimal solution

Notice that in the GVRP optimal solution the vehicles travel 41 additional miles than in the VRP optimal solution. However, weighted miles are 360 ton-miles less. Each ton-mile consumes around 3,350 BTU [9]. In addition, a gallon of diesel generates approximately 129,500 BTU [10]. Therefore, saving 360 ton-

miles saves approximately 9.3 gallons of diesel. Moreover, consuming one gallon of diesel generates 10.1 kg of CO₂. That is, the GVRP optimal solution saves 9.3 gallons of fuel, avoiding the emission of 93.9 kg. (i.e., 205.3 lb) of CO₂ to the atmosphere.

4. PROBLEM DIFFICULTY

As it was mentioned before, the GVRP is an extension of the traditional CVRP. Consequently, the GVRP is harder to solve since it adds a new set of variables and more constraints. The VRP formulation has $L(L+1)$ variables while the GVRP has $L(2L+1)$ variables. Also, the number of constraints for the VRP is $L(2L+1)$, while the number of constraints for the GVRP is $4L^2$. In order to illustrate the effect of the problem complexity some test problems were solved to optimality using IBM ILOG CPLEX Optimization Studio 12.2. Table 2 shows the results of the experiment. Notice that distances are in miles, weighted distances in ton-miles, and time in seconds.

L	K	VRP					GVRP				
		Var.	Const.	Dist	Wdist	Time	Var.	Const.	Dist	Wdist	Time
6	2	42	42	203	2455	0.05	72	144	207	2307	0.05
11	2	132	132	317	4442	3.43	242	484	320	3899	109.8
11	3	132	132	325	4063	1.39	121	484	333	3810	10.92
16	2	272	272	403	6985	0.22	512	1024	476	5635.5	7268.43*
16	3	272	272	501	7404	0.58	512	1024	507	7082	1165.22
16	4	272	272	432	7551	1.89	512	1024	443	6898	274.67

CPLEX solution times for GVRP are considerably larger than CPLEX solution times for VRP. In fact, CPLEX was interrupted for the GVRP with 16 locations and 2 vehicles after 121 minutes of computational time. Therefore that solution cannot be guarantee as optimal. Also, it is important to mention that, in most cases of GVRP, increasing the number of vehicles (without changing L) reduces computational time. That can be explained considering that the average number of customer assigned per vehicle is smaller, making the problem easier (i.e., smaller TSPs). Finally notice the minimizing distance traveled by vehicles does lead to higher weighted distances, which translate in higher CO₂ emissions.

5. LOCAL SEARCH

Since the GVRP is computationally hard, approximation algorithms are required to find good solutions in acceptable computational times. Approximation meta-heuristic algorithms such as tabu search and simulated annealing uses LS algorithms to explore the solution space when searching for good solutions. LS consists of a solution representation and one or more mechanisms to explore the solution space. A solution for the GVRP can be represented as a set of vectors $S = \{[s_1, s_i, \dots, s_j, s_1]_1 [s_1, s_n, \dots, s_m, s_1]_2 \dots [s_1, s_o, \dots, s_p, s_1]_K\}$, where s_1 represents the depot, and s_i, s_j, s_n, s_m, s_o and s_p are customer locations. Notice that each vector represents a route that starts and end at the depot. For example, a possible solution for the GVRP instance discussed above is $S = \{[1, 9, 7, 10, 2, 1]_1 [1, 5, 8, 6, 3, 11, 4, 1]_2\}$. Initial solutions are generated using construction algorithms. A construction algorithm for the GVRP is to assign locations to routes randomly without exceeding vehicle capacities. It is important to mention that generating an initial feasible solution (i.e., assigning all locations to routes without exceeding vehicle capacities) can be

challenging since the construction algorithm is solving a BPP. Notice that S representation can be simplified by removing the depot from the solution since all routes start and end at the depot. The simplified solution is $S = \{[9, 7, 10, 2]_1 [5, 8, 6, 3, 11, 4]_2\}$. The OF value (OFV) for S is 7,301 ton-miles with vehicle 1 initial cargo of 11 tons of cargo and vehicle 2 initial cargo of 12 tons.

Some of the most common moves used to exploring solution neighborhoods are 1^{opt} and 2^{opt} . The 1^{opt} mechanism simply takes one location in S and moves it to a different position in any route. An example of a 1^{opt} move is to remove location 11 from route 2 and to insert it in route 1 between locations 2 and 1 (i.e., after location 2). The new solution is $S_1 = \{[9, 7, 10, 2, 11]_1 [5, 8, 6, 3, 4]_2\}$ with an OFV of 7,156 ton-miles with vehicle 1 initial cargo of 12 tons and vehicle 2 with an initial cargo of 11 tons. Similarly, in a 2^{opt} move, two locations exchange their positions. Exchanging locations 2 and 6 in S_1 will lead to $S_2 = \{[9, 7, 10, 6, 11]_1 [5, 8, 2, 3, 4]_2\}$ is an example of a 2^{opt} move. The OFV for S_2 is 6,496 ton-miles with vehicle 1 initial cargo of 10 tons and vehicle 2 with an initial cargo of 13 tons. Notice that the solution is infeasible since the maximum capacity of each vehicle is 12 tons of cargo. In many cases accepting infeasible solutions is recommended when they provide improvements to OFV. In fact, a 1^{opt} move applied on S_2 that inserts location 4 between locations 6 and 11 lead to $S_3 = \{[9, 7, 10, 4, 6, 11]_1 [5, 8, 2, 3]_2\}$ which is the optimal solution for the problem instance.

A LS algorithm applies all possible moves to a given solution S and selects the move that provides the largest improvement over S solution. Then, the move is applied on S and the process is repeated for the new solution. The process stops when LS is not able to find a move that improves the quality of S .

6. CONCLUSIONS

This work presents an extension of the well known VRP called GVRP. The GVRP introduces a new objective function that minimizes weighted distance. Minimizing weighted distance reduces fuel consumption and consequently, CO₂ emissions. Moreover, this research introduces a mathematical formulation for the problem and a LS to find local optima. In addition, the problem is illustrated using a small instance and the GVRP difficulty is briefly discussed. Finally, areas of future research includes generating data sets for the GVRP; and developing more powerful meta-heuristics such as path re-linking, simulated annealing, and tabu search. These meta-heuristics are necessary to find good solutions to real size problems in acceptable computational time.

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