

A GENERALIZATION OF THE RELAXED CHOICE OF TECHNOLOGY MODEL

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ABSTRACT

The relaxed choice of technology model in [1] is generalized. The generalization ensures that pollutants generated to satisfy both internal and external demands are controlled by choosing the right set of technologies.

INTRODUCTION

The amount of pollutants produced when an item is manufactured depends on the raw material and the available technologies [1, 4]. The model in [1] shows that when an industry uses the best available raw materials for production, technology becomes a major factor in reducing industrial pollution. The objective of this paper is to generalize the relaxed model given in [1] so that the model could control the generation of pollutants needed for both internal and external demands.

The structure of the rest of the paper is as follows. In Section 2, the relaxed choice of technology model is presented. In Section 3, a generalization of the model is provided. An algorithm for solving the modified model is given in Section 4. In Section 5, we summarize our results.

THE RELAX TECHNOLOGY MODEL

Consider an economy with n sectors each of which can produce at least one output. There are interactions among different sectors as an output from one sector can be used by other sectors for their production. As in the classical input-output model [4], we assume that prices are fixed and that demand quantities are stable. One period of activities will be considered.

The problem of interest is as follows: Given different technologies for the production of an item by sector j , $j = 1, \dots, n$, which technology should be chosen by sector j so as to (1) satisfy permissible pollutant level for the sector, and (2) satisfy demand for sector j 's output as much as possible.

Let

n = number of sectors in the economy

x_j = maximum amount of pollutants produced by sector j

- m_j = number of different technologies available for the production of an output by sector j. Assume $m_j \geq 1, j = 1, \dots, n$
- b^j = maximum amount of pollutants that sector j should produce in order to satisfy external demands for its services
- a_{ik}^j = units of output of pollutants by sector j using technology i needed by sector k to produce one unit of its pollutants

Define the following matrices and vectors:

$$A^j = (a_{ik}^j), \quad i = 1, \dots, m_j, \quad k = 1, \dots, n, \quad j = 1, \dots, n.$$

$$E^j = \text{an } m_j \times n \text{ matrix with 1s in column } j \text{ and zeros in all other columns.}$$

$$A = \begin{bmatrix} A^1 \\ \bullet \\ \bullet \\ \bullet \\ A^n \end{bmatrix}, \quad E = \begin{bmatrix} E^1 \\ \bullet \\ \bullet \\ \bullet \\ E^n \end{bmatrix}, \quad Q^j = \begin{bmatrix} b^j \\ \bullet \\ \bullet \\ \bullet \\ b^j \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ \bullet \\ \bullet \\ \bullet \\ x_n \end{bmatrix}$$

If we set $m = \sum_{j=1}^n m_j$, then A^j is $m_j \times n$, A is $m \times n$, $m \geq n$, Q^j is $m_j \times 1$, and X is $n \times 1$.

The Relaxed Model

The condition that the total amount of pollutants generated by sector j using technology i is equal to the amount required to satisfy inter-sector and external demands is equivalent to

Technology Choice Model (TCM):

$$(E^j - A^j)_i X \leq Q_i^j, \quad j = 1, \dots, n, \quad (1)$$

$$X \geq 0$$

where $i \in (1, \dots, m_j)$ and A_i^j = row i of matrix A . We note that $(E^j - A^j)_i X \geq 0$.

A GENERALIZATION OF THE RELAXED MODEL

The model in Section 2 is generalized in this section. The assumptions of the relaxed model apply.

Let p^j be the amount of pollutants that sector j should produce in order to satisfy internal demands for its goods and services; as well as meet emission restrictions. Define the vector P^j by

$$P^j = \begin{bmatrix} p^j \\ \cdot \\ \cdot \\ \cdot \\ p^j \end{bmatrix}$$

where $j = 1, \dots, n$.

The condition that the pollutants generated by sector j using technology i is at most equal to the amount produced for both internal and external demands is given by:

Generalized Relaxed Technology Choice Model (GRTCM):

$$(E^j - A^j)_i X \leq Q_i^j, \quad j=1, \dots, n, \quad (2)$$

$$A_i^j X \leq P_i^j \quad (3)$$

$$X \geq 0$$

where $i \in (1, \dots, m_j)$ and A_i^j = row i of matrix A .

SOLVING THE GRTCM

We consider solving the GRTCM in this section. Each sector j has m_j technologies to choose from for producing an item. Therefore, the total number of different combination of technologies available for the economy is $T = \prod_{j=1}^n m_j$. The problem can be solved by solving $T = \prod_{j=1}^n m_j$ system of linear inequalities.

However, the condition $X \geq 0$ makes it difficult to solve it efficiently using this approach. We will use a linear programming approach.

Using a linear program

Solving the model by a linear program is preferable since that will take care of the non-negativity condition on the variables. Moreover, the use of linear programs allows for sensitivity analysis, a handy tool when there are changes in the pollution input – output coefficients or in the required pollutant levels. However, there is one problem with using a linear program. The linear programming approach requires an objective function, which is not available in the GRTCM model. This is easy to get around as given in the algorithms below.

Let S be a technology state of the economy. Then $S = S(i_1, \dots, i_n)$ specifies the technology composition of the economy; where i_j means that sector j uses technology i in the production of its outputs. For each j , define

$\alpha_i^j =$ amount by which technology i of sector j under estimates the total amount of pollutants that should be produced by sector j in order to satisfy external demands

$\delta_i^j =$ amount by which technology i of sector j under estimates the total amount of pollutants that should be produced by sector j in order to satisfy internal demands

Define the following vectors:

$$\alpha^j = (\alpha_i^j), \quad i=1, \dots, m_j,$$

$$\delta^j = (\delta_i^j), \quad i=1, \dots, m_j,$$

where $j=1, \dots, n$.

Algorithm 1

1. For each technology state $S = S(i_1, \dots, i_n)$, solve the linear program $LP_1(S)$:

$$\begin{aligned} \text{Min } z &= \sum (\alpha_{i_j}^j + \delta_{i_j}^j) \\ \text{St. } (E_{i_j}^j - A_{i_j}^j)X + \alpha_{i_j}^j &= Q_{i_j}^j \end{aligned} \tag{4}$$

$$A_i^j X + (\delta_{i_j}^j) = P_i^j \tag{5}$$

$$X \geq 0, \alpha_{i_j}^j \geq 0, \delta_{i_j}^j \geq 0.$$

where $i_j \in S(i_1, \dots, i_n)$, $i = 1, \dots, m_j$, $j = 1, \dots, n$.

2. If $LP_1(S)$ has a solution for any j , $j \in (1, \dots, n)$, with $z = 0$, then the solution solves the GRTCM. Otherwise, the GRTCM has no solution.

The following theorem shows that Algorithm 1 solves the GRTCM.

Theorem 1

If the $LP_1(\bar{S})$ has an optimal solution with objective function value $z(\bar{S}) = 0$, then the technology state $\bar{S}(i_1, \dots, i_n)$ satisfies the pollutants emission requirement for the economy.

Proof: Suppose that there is a technology state $\bar{S}(i_1, \dots, i_n)$ such that the $LP_1(\bar{S})$ has an optimal objective function value $z(\bar{S}) = 0$. Then we have that $\sum (\alpha_{i_j}^j + \delta_{i_j}^j) = 0$. Since $\alpha_{i_j}^j \geq 0$, $\delta_{i_j}^j \geq 0$, we have that for each j , $\alpha_{i_j}^j + \delta_{i_j}^j \geq 0$. Thus $z(\bar{S}) = 0$ implies $\alpha_{i_j}^j = 0$, $\delta_{i_j}^j = 0$. That is, there is no under production of pollutants when $\bar{S}(i_1, \dots, i_n)$ is applied to the economy. So $\bar{S}(i_1, \dots, i_n)$ satisfies the pollution emission requirement of the economy. This completes the proof.

For each j , the pollutant quantity generated by the chosen technology gives exactly the amount that sector j should produce.

Algorithm 2

Consider the LP2:

$$\max \quad z = e^t X$$

$$\text{Subject to:} \quad (E - A)X \leq Q$$

$$AX \leq P, \quad X \geq 0$$

where $e = (1, \dots, 1)$ is an $n \times 1$ column vector.

If $\bar{X}$ solves the LP2, then $\bar{X}$ satisfies $(E - A)\bar{X} \leq Q$, $A\bar{X} \leq P$, $\bar{X} \geq 0$, the system for the GRTCM. This observation justifies the following steps for solving the GRTCM.

Step 1: Determine all the technology composition of the economy.

Step 2: For each technology composition $S(i_1, \dots, i_n)$, solve the linear program:

LP(S): $\max z = e^t X$

Subject To $(E^j - A^j)_{i_j} X \leq Q_{i_j}, i \in (1, \dots, m_j), j = 1, \dots, n$

$X \geq 0$

Step 3: If there exists a $j \in \{1, \dots, n\}$ such that LP(S) has a solution $\bar{X}$ with binding constraints, then $\bar{X}$ solves the GRTCM. Otherwise, the GRTCM has no solution.

The technology chosen corresponds to the rows with binding constraints.

Example

Consider an economy consisting of two sectors: auto and steel. Suppose that each sector has two technologies to choose from for producing steel and cars. Assume that in this economic system, the air pollution input-output coefficients and demand quantities are as given in Table 1.

Table 1				
	Auto		Steel	
Technology 1	0.25	1.00	0.056	0.12
Technology 2	0.24	0.98	0.055	0.14
Pollution for external demands (tons)	275		60	
Pollution for internal demands (tons)	225		40	

Table 1: Air pollution input-output coefficients

We want to select appropriate set of technologies that each sector can use to produce its products without exceeding the permissible levels.

Solution:

The technology states are S(1, 1), S(1, 2), S(2, 1), S(2, 2), where, for example, S(1, 2) means that the auto sector uses technology 1 and the steel sector uses technology 2 for their productions.

For technology state S(1, 1), we have

$$A = \begin{bmatrix} 0.250 & 1.000 \\ 0.056 & 0.120 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E - A = \begin{bmatrix} 0.750 & -1.000 \\ -0.056 & -0.880 \end{bmatrix}, \quad P = \begin{bmatrix} 275 \\ 60 \end{bmatrix}, \quad P = \begin{bmatrix} 225 \\ 40 \end{bmatrix}$$

Solving the GRTCM by Algorithm 1 or Algorithm 2, we obtain $X = (500, 100)$, where the answers may vary due to truncation errors. The rest of the solutions are given in Table 2.

Table 2				
	S(1, 1)	S(1, 2)	S(2, 1)	S(2, 2)
(x_1, x_2)	(500, 100)	(490.625, 92.9682)	(489.9661, 99.3615)	484.7162, 95.290085)
$\begin{bmatrix} E - A \\ A \end{bmatrix} X - \begin{bmatrix} Q \\ P \end{bmatrix}$	(0,0,0,0)	(0, -7.0313, -9.3750, 0)	(0, 0, -10.0339, -0.6385)	(0, -4.7099, -15.2838, 0)

Table 2: Solution to all technology states

The solution corresponding to the technology state S(1, 1) is the one that satisfies the permissible pollutant levels and also has binding constraints. The binding constraints ensure that the sectors satisfy all their demands. Hence, the auto sector should use technology 1 and the steel sector technology 1, respectively, in their productions.

CONCLUSION

The relaxed input-output pollution model formulated in [1] is generalized. The generalized model controls pollutant quantities needed for internal and external demands. The method is based on the input-output pollution coefficients.

The selection of appropriate set of technologies is done by solving the model using a linear program. The use of linear program makes it possible to perform sensitivity analysis when problem parameters need to be changed.

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